Math Challenge
Washington Middle School
March 4, 2010

Warm-Up Activity

Pretend that it's 3:00 now. Answer the following questions, but don't worry about AM/PM.

Problem 1a: In 17 hours, what time will the clock show?

Problem 1b: In 33 hours, what time will the clock show?

Problem 1c: What time did the clock show 15 hours ago?

Problem 1d: What time will the clock read 17 hours after the time it shows 19 hours before 4:00?

Today is Thursday. Answer the following questions.

Problem 2a: What day of the week will it be 5 days from now?

Problem 2b: What day of the week will it be 17 days from now?

Problem 2c: What day of the week was it 10 days ago?
We use the notation $A = R \ (\text{mod } N)$ to mean that $A$ has remainder $R$ when divided by $N$.

**Problem 3:** Fill in the blanks.

a) $8 \ (\text{mod } 3) = \underline{2}$

b) $17 \ (\text{mod } 8) = \underline{1}$

c) $8 \ (\text{mod } 4) = \underline{0}$

d) $7 \ (\text{mod } 5) = \underline{2}$

e) $6 \ (\text{mod } 5) = \underline{1}$

1. $7 + 6 \ (\text{mod } 5) = \underline{4}$

2. $2 + 1 \ (\text{mod } 5) = \underline{3}$

f) $20 \ (\text{mod } 11) = \underline{9}$

g) $35 \ (\text{mod } 11) = \underline{1}$

1. $20 + 35 \ (\text{mod } 11) = \underline{4}$

2. $9 + 2 \ (\text{mod } 11) = \underline{1}$

h) $7 \ (\text{mod } 3) = \underline{1}$

i) $5 \ (\text{mod } 3) = \underline{2}$

1. $7 \times 5 \ (\text{mod } 3) = \underline{1}$

j) $5 \ (\text{mod } 4) = \underline{1}$

1. $5 \times 5 \ (\text{mod } 4) = \underline{1}$

2. $5 \times 5 \times 5 \ (\text{mod } 4) = \underline{1}$

k) $4 \ (\text{mod } 5) = \underline{4}$

1. $4 \times 4 \ (\text{mod } 5) = \underline{1}$

2. $4 \times 4 \times 4 \ (\text{mod } 5) = \underline{1}$

3. $4 \times 4 \times 4 \times 4 \ (\text{mod } 5) = \underline{1}$
**Problem 4:** What is the remainder of $2007 \times 2008 + 2009^2$ when divided by 7?

**Problem 5:** If your birthday was on a Tuesday last year, on what day will your birthday fall this year? On what day did your birthday fall the previous year?

**Problem 6:** Pretend you were born on March 2. In 2003, your birthday was on a Monday. On what day did your birthday fall in 2004?

**Problem 7:** On what day of the week were you born?
Problem 8: Harry goes to the store to buy some candy. He buys:

- 24 kit-kats
- 17 peanut butter cups
- 16 snickers.

Snickers and peanut butter cups cost the same price and the cashier charges Harry $18.65. Harry realizes that the cashier made a mistake and turns him into a toad. How did he know?

Problem 9:* What are the last two digits of $2^{2010}$?

Hint: Compute the last two digits of the following numbers:

- $2^1$
- $2^2$
- $2^4$
- $2^8$
- $2^{16}$
- $2^{32}$
- $2^{64}$
- $2^{128}$
- $2^{256}$
- $2^{512}$
- $2^{1024}$
Problem 10: Jeff adds 3 counting numbers \((w + x + y)\) and correctly gets an even sum. Karen adds 2 of the same numbers as Jeff added, plus a different third number \((w + x + z)\) and correctly gets an odd sum. Is the sum of \(y + z\) even or odd?

Problem 11*: What is the last digit of the number

\[1^2 + 2^2 + 3^2 + \ldots + 98^2 + 99^2?\]
Challenge Problems

**CP1**: Let $N = a_m a_{m-1} ... a_2 a_1 a_0$ be an $m$-digit number with digits $a_0, a_1, ..., a_m$.

1. Show that $N$ is divisible by 3 if and only if $a_0 + a_1 + ... + a_m$ is divisible by 3.
2. Show that $N$ is divisible by 9 if and only if $a_0 + a_1 + ... + a_m$ is divisible by 9.
3. Show that $N$ is divisible by 11 if and only if $a_0 - a_1 + a_2 - ... \pm a_m$ is divisible by 11.

**CP2**: Show that $2222^{5555} + 5555^{2222}$ is divisible by 7.

**CP3**: Show that $3^n - 2^n$ is divisible by 35 for any positive integer $n$. 