Math Challenge

Washington Middle School
November 12, 2009

Maps and Graphs

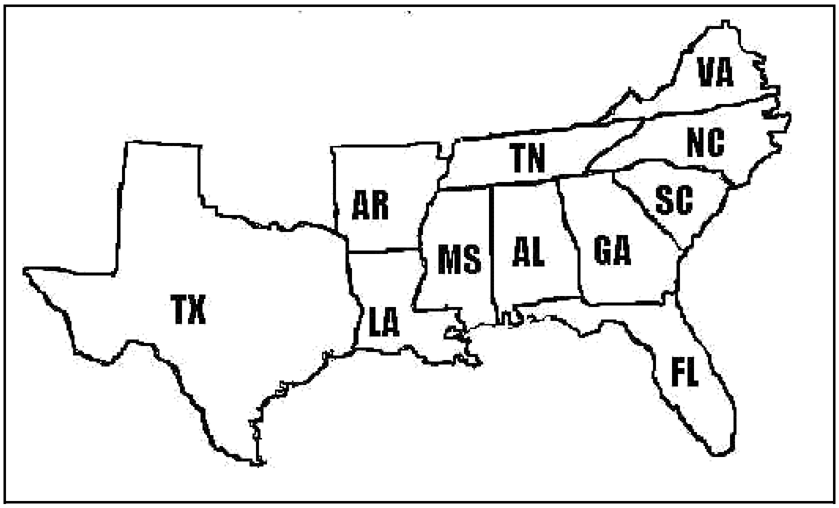
Warm-up problem 1: Ten swimmers have a race. How many different orders can they finish in?

Warm-up problem 2: The gold, bronze, and silver medalists stand on a podium to get their awards. How many different combinations of winners are there out of the ten swimmers? (Note: if Anton wins the gold, Steve wins the silver, and Toby wins the bronze, this counts as a different combination as if Toby wins the gold, Steve wins the silver, and Anton wins the bronze.)

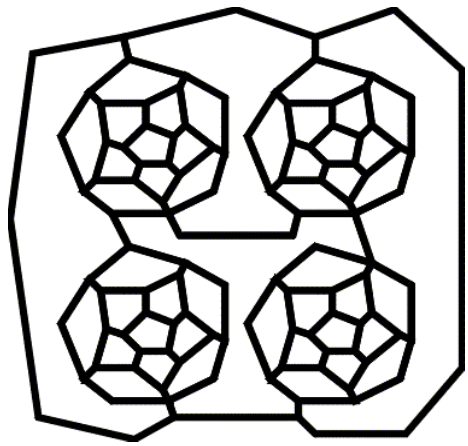
Warm-up problem 3: Two of the ten swimmers are from Uzbekistan. If all the swimmers have the same chance of winning, what's the probability that both of the Uzbek swimmers finish in the top three?

Instructions:

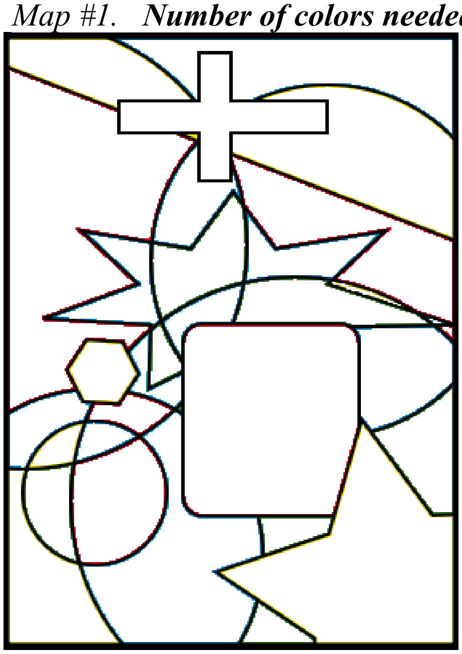
Exercise 1. With a partner, take a look at the following maps. Try to color in each map using as few colors as possible, but with one condition: if two regions share a border, you can't give them the same color! (If the countries don't share a border but just touch at a single point, it's okay to give them the same color.) Use colored pencils, but make sure you're using as few colors as possible before you start drawing—you can write in a color in pencil at first. Under each map, write down how many colors you needed for it.



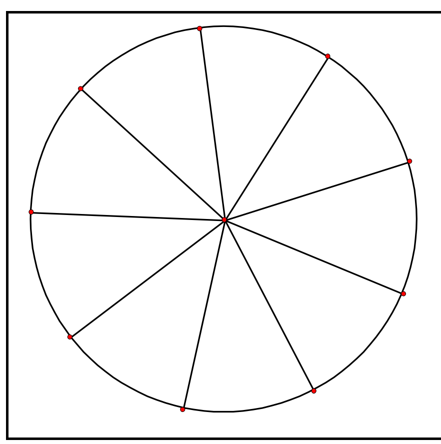
Map #1. ***Number of colors needed:***



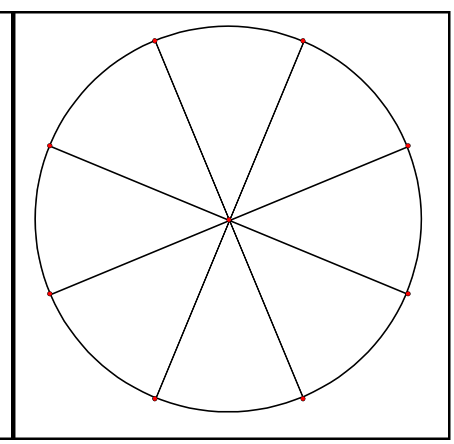
Map #2. ***Number of colors needed:***



Map #3. ***Number of colors needed:***



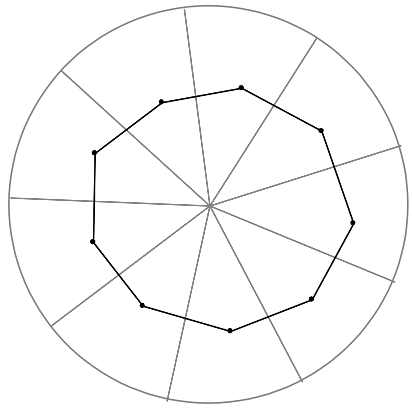
Map #4. ***Number of colors needed:***



Map #5. ***Number of colors needed:***

What is the smallest number of colors you needed to color one of these maps? What is the largest number of colors you needed to color one of these maps? What do you think is the most number of colors a map might need?

Exercise 2. Now, we're going to take each map and make a new diagram out of it. In the middle of each region, make a point. Then, draw a line between points if they're in neighboring regions. For example, from map #4, we get



Do the same for map #1:

This new object is called a *graph*. The points are called *vertices* and the lines connecting them are called *edges*.

Exercise 3. For the two graphs above, give each vertex a color, following this rule: if two vertices are connected by an edge, they must be colored differently. Try to use as few colors as possible. How many colors do you need for each of these two graphs?

Exercise 4. Can you draw a graph that can't be colored with three colors? How about one that can't be colored with four colors? And one that can't be colored with five colors?

Exercise 5. Say you have four vertices, and you decide to connect *all* of them by edges. How many edges do you have? How many edges are there if you have five vertices and connect them all? Six vertices? What about *n* vertices?

Exercise 6. You run trains between 10 different cities in Washington. From each city, there are train routes to five of the other cities. Prove that if you're allowed to transfer from one train to another, you can get from any of the ten cities to another.

Exercise 7. Again, you run trains between 10 different cities in Washington. This time, all you know is that there are a total of 37 train routes between cities. Each route goes in both directions—that is, if a train goes from Seattle to Tacoma, it also goes from Tacoma to Seattle. Can you get from any of the ten cities to another?

Exercise 8. Seattle has decided to expand the light rail! Can you make tracks from each neighborhood to each other neighborhood without any of the tracks crossing?