Math Challenge

Washington Middle School
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Mathematical Strategy Games

Instructions: Play the following games with a partner. Try to figure out if there is a strategy that will guarantees a certain player can always win.

Game 0:
You will start with a pile of N coins on the table. Players take turns removing one coin from the pile, and the player who takes the last coin wins.

Play this game a few times with your partner. Keep track of who goes first and who wins each time.
  a. Start with a pile of three coins.

      Name of Player 1:

      Name of Player 2:

      Who won?

  b. Start with a pile of four coins.

      Name of Player 1:

      Name of Player 2:

      Who won?

  c. Start with a pile of five coins.

      Name of Player 1:

      Name of Player 2:

      Who won?

  d. Do you see a pattern? Who will win if you start with a pile of N coins? Why?

Let's try something more complicated.

Game 1:
Start with a pile of N coins on the table. Each player can remove either one coin or two coins from the pile on his or her turn. The player who takes the last coin wins.

Play this game a few times with your partner. Keep track of who goes first and who wins each time.

    a. Start with a pile of two coins.

        Name of Player 1:

        Name of Player 2:

        Who won? Can this person always win? How?

    b. Start with a pile of four coins.

        Name of Player 1:

        Name of Player 2:

        Who won? Can this person always win? How?

    c. Start with a pile of five coins.

        Name of Player 1:

        Name of Player 2:

        Who won? Can this person always win? How?

    d. Start with a pile of three coins.

        Name of Player 1:

        Name of Player 2:

        Who won? Can this person always win? How?

e. Start with a pile of seven coins.

Name of Player 1:

Name of Player 2:

Who won? Can this person always win? How?

f. Start with a pile of N coins. When does Player 1 win? When does Player 2 win? How?

Game 2:

This is another game for two players. Start by drawing M 1's on the page and N 0's on the page. On your turn, pick two numbers to cross out. If the numbers are the same, put a 0 on the page. If the numbers are different, put a 1 on the page. (So on your turn, you cross two numbers off the page and add one number to the page). Play the game until there is only one number left on the page. If this number is a 1, Player 1 wins. If it is a 0, Player 2 wins.

a. Start with 6 1's and 5 0's. Play the game a few times with your partner, keeping track of who wins each time.

Name of Player 1:

Name of Player 2:

Who won?

b. Start with 5 1's and 3 0's. Play the game a few times with your partner, keeping track of who wins each time.

Name of Player 1:

Name of Player 2:

Who won?

c. Let's be more methodical about the game. Play again with 6 1's and 5 0's.

After each turn, keep track of the sum of all the numbers that haven't been crossed out. Do you notice anything?

Turn Number	Numbers on the board	Sum
0	1,1,1,1,1,1,0,0,0,0,0	6
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		

d. If you play the game, starting with N 1's and M 0's, who will win? Why?

Game 3:

Pick a number, N, and write all of its divisors on the paper. On your turn, pick a divisor D of N. Cross all the multiples of D off the list, including D itself. Players take turns doing this. The player who is forced to choose the divisor D=1 loses.

Here's an example. Let's say N=12. The divisors of 12 are:

$$1 \qquad 2 \qquad 3 \qquad 4 \qquad 6 \qquad 12$$

Alice and Bob will play the game. Alice goes first.

Alice chooses the divisor D=3. She crosses the numbers 3, 6, and 12 from the list.

Bob then chooses the divisor D=2. He crosses the numbers 2 and 4 from the list.

The only remaining number on the list is 1. Alice loses!

Play this game with your partner. Pick your favorite numbers A, B, and C that are smaller than 200.

a. Start with the number A and all of its divisors. Play the game three times with your partner, keeping track of who goes first and who wins.

b. Start with the number B and all of its divisors. Play the game three times with your partner, keeping track of who goes first and who wins.

c. Start with the number C and all of its divisors. Play the game three times with your partner, keeping track of who goes first and who wins.