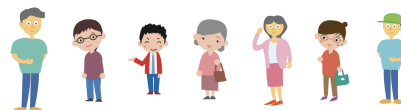# University of Washington Math Olympiad, 2025

Grades 8–10

Problem #1   There were 2025 people standing in a line. The order in the line got mixed up a bit, but each person ended up within one spot of their original position. (For example, the person who started in the 6th position ended up in either 5th, 6th, or 7th position.) Show that at least one person ends up in their original position in line.
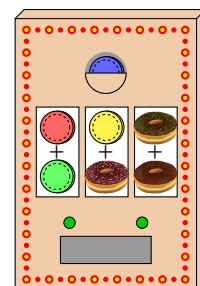
Problem #2   There are 11 evenly spaced houses on Elm Street. Each house is either red or blue. Show that there are three houses of the same color spaced equally apart.

Problem #3   Sylvio discovered a token for a magical donut machine. He inserted the token into the machine and received a donut along with another token. Each time he inserts a token, the machine gives him a reward: either two donuts, a donut and a token, or two tokens.

Sylvio eats any donuts he receives and continues inserting tokens into the machine. He ate a total of 2025 donuts and did not have anymore tokens left. How many tokens did Sylvio insert?

Problem #4   Lily and Mark are playing a game. Each of them has an army of soldiers. They are competing to capture 4 castles worth 1, 2, 3, and 4 points, respectively. The players secretly send all of their soldiers to the castles. Each soldier goes to exactly one castle.

The player who sends more soldiers to a castle captures it. In case of a tie, nobody captures the castle. In the example below, Mark wins 5 points by capturing castles A and D. Lily wins 3 points by capturing castle C.

Lily has 10 soldiers in her army. What is the smallest number of soldiers Mark needs in his army to guarantee he can win at least 6 points, no matter how Lily places her army?

	A 1 pt	B 2 pts	C 3 pts	D 4 pts
Lily:	2	1	(5)	0
Mark:	(4)	1	2	(7)

Problem #5   In the faraway land of Artinia, all roads are one-way roads. According to the law of the land, for any two cities A and B, if there is a one-way road from A to B, then a one-way road from B to A is not allowed.

The Ministry of Transport wants a budget to build one more road. They notice that no matter where the road is built, it will be possible to drive from any city to any other city. Prove that it is already possible to drive between any pair of cities in Artinia without adding the new road.