

University of Washington Math Olympiad, 2025

Grades 8–10

Problem #6 Nina loves numbers that start with 9. She picked 20 distinct numbers between 900 and 999. She then wrote them all down from smallest to largest without spaces to form a 60-digit number, for example 900907...985994.

Prove that Nina's 60-digit number is not divisible by 91.

Problem #7 There are 100 Wonka chocolates distributed in 10 cups, which are arranged as shown in the picture below. Each cup contains at least 7 chocolates.

Augustus wants to eat all of the chocolates, but he must follow Wonka's rules:

- He first must choose a cup with an even number of chocolates.
- He then must move 1 chocolate from this chosen cup to each cup that it is touching.
- Finally he must remove the chosen cup from the table and eat the remaining chocolates in the cup.

Augustus can repeat this process as long as at least one remaining cup has an even number of chocolates.

Prove that Augustus will not be able to eat all of the chocolates.

