# University of Washington Math Olympiad, 2025

## Grades 6–7

**Problem #1** There were $11$ people standing in a line. The order in the line got mixed up a bit, but each person ended up within one spot of their original position. (For example, the person who started in the $6$th position ended up in either $5$th, $6$th, or $7$th position.) Show that at least one person ends up in their original position in line.

**Problem #2** There are $19$ people standing in a circle, facing the center. Each person is either a knight or a liar. Knights always tell the truth and liars always tell lies. Each person says, "Out of the four people to the right of me, at least two are knights." How many knights could there be in the circle?

**Problem #3** A piece of graph paper extends forever in all directions. Each square contains one of the four digits $1$, $2$, $3$, or $4$. Each digit is used at least once. Two squares are called *neighbors* if they share a side. A square is called *balanced* if its digit is equal to the number of different digits in its four neighbors. (In the example shown, the blue square is balanced but the orange one is not. Both have three different digits as neighbors.) Is it possible that all squares can be balanced?

| | | | |
|---|---|---|---|
| | | 1 | |
| | 3 | 2 | 1 |
| 1 | 3 | 4 | |
| | 4 | | |

**Problem #4** Ten tennis players compete in a tournament that lasts $9$ days. Each person plays one game per day. The winner of each game gets $1$ point, and the loser gets $0$ points. Ties are not allowed.

After $5$ days, Daphne is in second place: exactly one player has more points than she does, and the rest of the players have fewer points than Daphne. Prove that it is impossible for Daphne to have fewer points than each of the other players at the end of the tournament.

**Problem #5** Lily and Mark are playing a game. Each of them has an army of soldiers. They are competing to capture $5$ castles. The players secretly send all of their soldiers to the castles. Each soldier goes to exactly one castle. The player who sends more soldiers to a castle captures it. In case of a tie, nobody captures the castle. In the example shown, Mark captures castles A and D, Lily captures castle C, and nobody captures castle B or E.

Lily has $10$ soldiers in her army. What is the smallest number of soldiers Mark needs in his army to guarantee he can capture at least $3$ castles, no matter how Lily places her army?