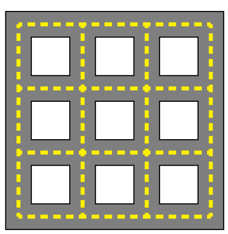
# University of Washington Math Hour Olympiad 2022

## Grades 6–7

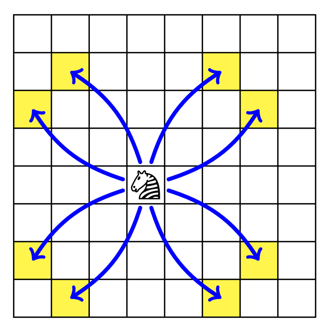
**1.** Nineteen witches, all of different heights, stand in a circle around a campfire. Each witch says whether she is taller than both of her neighbors, shorter than both, or in-between. Exactly three said "I am taller." How many said "I am in-between"?

**2.** Alex is writing a sequence of A's and B's on a chalkboard. Any $20$ consecutive letters must have an equal number of A's and B's, but any $22$ consecutive letters must have a different number of A's and B's. What is the length of the longest sequence Alex can write?

**3.** A police officer patrols a town whose map is shown. The officer must walk down every street segment at least once and return to the starting point, only changing direction at intersections and corners. It takes the officer one minute to walk each segment. What is the fastest the officer can complete a patrol?

**4.** A *zebra* is a new chess piece that jumps in the shape of an "L" to a location three squares away in one direction and two squares away in a perpendicular direction. The picture shows all the moves a zebra can make from its given position. Is it possible for a zebra to make a sequence of $64$ moves on an $8\times 8$ chessboard so that it visits each square exactly once and returns to its starting position?



**5.** Ann places the integers $1, 2, \ldots, 100$ in a $10\times 10$ grid, however she wants. In each round, Bob picks a row or column, and Ann sorts it from lowest to highest (left-to-right for rows; top-to-bottom for columns). However, Bob never sees the grid and gets no information from Ann.

After eleven rounds, Bob must name a single cell that is guaranteed to contain a number that is at least $30$ and no more than $71$. Can he find a strategy to do this, no matter how Ann originally arranged the numbers?

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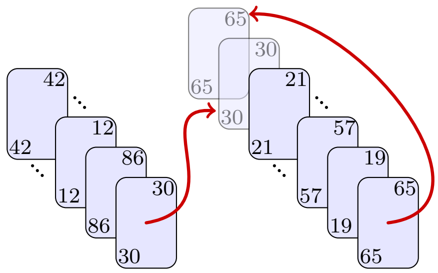
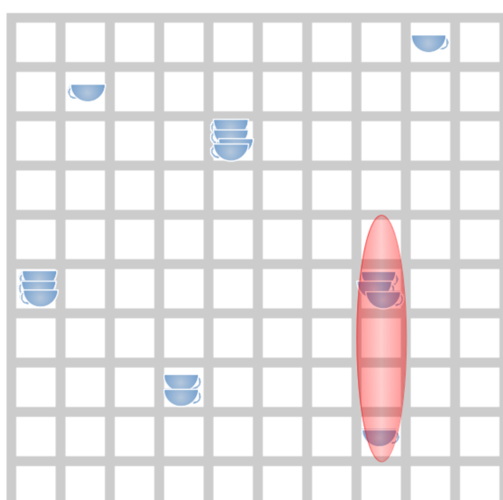
**6.** Evelyn and Odette are playing a game with a deck of $101$ cards numbered $1$ through $101$. At the start of the game the deck is split, with Evelyn taking all the even cards and Odette taking all the odd cards. Each shuffles her cards. On every move, each player takes the top card from her deck and places it on a table. The player whose number is higher takes both cards from the table and adds them to the bottom of her deck, first the opponent's card, then her own. The first player to run out of cards loses.

Card $101$ was played against card $2$ on the $10^{\text{th}}$ move. Prove that this game will never end.




**7.** The Vogon spaceship *Tempest* is descending on planet Earth. It will land on five adjacent buildings within a $10 \times 10$ grid, crushing any teacups on roofs of buildings within a $5 \times 1$ length of blocks (vertically or horizontally). As Commander of the Space Force, you can place any number of teacups on rooftops in advance. When the ship lands, you will hear how many teacups the spaceship breaks, but not where they were. (In the figure, you would hear $4$ cups break.)

What is the smallest number of teacups you need to place to ensure you can identify at least one building the spaceship landed on?

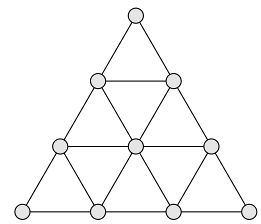


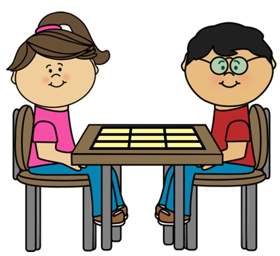
# University of Washington Math Hour Olympiad 2022

## Grades 8–10


**1.** Alex is writing a sequence of A's and B's on a chalkboard. Any $20$ consecutive letters must have an equal number of A's and B's, but any $22$ consecutive letters must have a different number of A's and B's. What is the length of the longest sequence Alex can write?

**2.** A positive number is placed on each of the $10$ circles in this picture. It turns out that for each of the nine little equilateral triangles, the number on one of its corners is the sum of the numbers on the other two corners. Is it possible that all $10$ numbers are different?





**3.** Pablo and Nina take turns entering integers into the cells of a $3 \times 3$ table. Pablo goes first. The person who fills the last empty cell in a row must make the numbers in that row add to $0$. Can Nina ensure at least two of the columns have a *negative* sum, no matter what Pablo does?

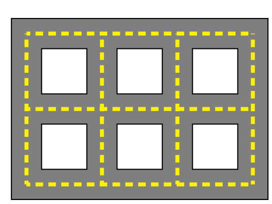
**4.** All possible simplified* fractions greater than $0$ and less than $1$ with denominators less than or equal to $100$ are written in a row with a space before each number (including the first).

Zeke and Qing play a game, taking turns choosing a blank space and writing a "$+$" or "$-$" sign in it. Zeke goes first. After all the spaces have been filled, Zeke wins if the value of the resulting expression is an integer. Can Zeke win no matter what Qing does?

$$\cdots \;\_\_\; \frac{21}{23} \; + \; \frac{22}{23} \;\_\_\; \frac{1}{24} \; + \; \frac{5}{24} \; - \; \frac{7}{24} \;\_\_\; \cdots$$

* Simplified means, for example, "$1/3$" is written but "$2/6$" is not.

**5.** A police officer patrols a town whose map is shown. The officer must walk down every street segment at least once and return to the starting point, only changing direction at intersections and corners. It takes the officer one minute to walk each segment. What is the fastest the officer can complete a patrol?



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## Grades 8–10

**6.** Prove that among any $3^{2022}$ integers, it is possible to find exactly $3^{2021}$ of them whose sum is divisible by $3^{2021}$.

**7.** Given a list of three numbers, a *zap* consists of picking two of the numbers and decreasing each of them by their average. For example, if the list is $(5,\ 7,\ 10)$ and you zap $5$ and $10$, whose average is $7.5$, the new list is $(-2.5,\ 7,\ 2.5)$.


Is it possible to start with the list $(3,\ 1,\ 4)$ and, through some sequence of zaps, end with a list in which the sum of the three numbers is $0$?