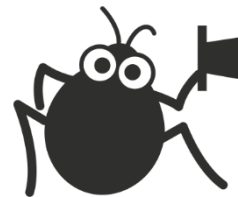


University of Washington Math Hour Olympiad, 2016

Grades 6-7

6. A circus owner will arrange 100 fleas on a long string of beads, each flea on her own bead. Once arranged, the fleas start jumping using the following rules. Every second, each flea chooses the closest bead occupied by one or more of the other fleas, and then all fleas jump simultaneously to their chosen beads. If there are two places where a flea could jump, she jumps to the right.



At the start, the circus owner arranged the fleas so that, after some time, they all gather on just two beads. What is the shortest amount of time it could take for this to happen?



7. The faraway land of Noetheria has 2016 cities. There is a nonstop flight between every pair of cities. The price of a nonstop ticket is the same in both directions, but flights between different pairs of cities have different prices. Prove that you can plan a route of 2015 consecutive flights so that each flight is cheaper than the previous one. It is permissible to visit the same city several times along the way.