

University of Washington Math Hour Olympiad, 2015

Grades 8-10

6. The sum of 2015 rational numbers is an integer. The product of every pair of them is also an integer. Prove that they are all integers.

(A rational number is one that can be written as m/n , where m and n are integers and $n \neq 0$.)

7. An $N \times N$ table is filled with integers such that numbers in cells that share a side differ by at most 1. Prove that there is some number that appears in the table at least N times.

For example, in the 5×5 table below the numbers 1 and 2 appear at least 5 times.

1	2	3	4	5
2	1	2	3	4
1	0	1	2	3
1	1	0	1	2
0	0	1	2	3