

University of Washington Math Hour Open Olympiad, 2012

Grades 5-7

6. In a galaxy far, far away, there is a United Galactic Senate with 100 Senators. Each Senator has no more than three enemies. Tired of their arguments, the Senators want to split into two parties so that each Senator has no more than one enemy in his own party. Prove that they can do this.
 (Note: If A is an enemy of B, then B is an enemy of A.)

7. Harry has a 2012×2012 chessboard and checkers numbered from 1 to 2012×2012 . Can he place all the checkers on the chessboard in such a way that whatever row and column Professor Snape picks, Harry will be able to choose three checkers from this row and this column such that the product of the numbers on two of the checkers will be equal to the number on the third?

