University of Washington Math Hour Open Olympiad, 2011
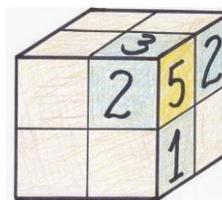
Grades 8-9

1. Twelve people, some are knights and some are knaves, are sitting around a table. Knaves always lie and knights always tell the truth. At some point they start up a conversation. The first person says, "There are no knights around this table." The second says, "There is at most one knight at this table." The third – "There are at most two knights at the table." And so on until the 12th says, "There are at most eleven knights at the table." How many knights are at the table? Justify your answer.

2. Show that in the sequence 10017, 100117, 1001117, … all numbers are divisible by 53.

3. Harry and Draco have three wands: a bamboo wand, a willow wand, and a cherry wand, all of the same length. They must perform a spell wherein they take turns picking a wand and breaking it into three parts – first Harry, then Draco, then Harry again. But in order for the spell to work, Harry has to make sure it is possible to form three triangles out of the pieces of the wands, where each triangle has a piece from each wand. How should he break the wands to ensure the success of the spell?

4. A $2 \times 2 \times 2$ cube has 4 equal squares on each face. The squares that share a side are called neighbors (thus, each square has 4 neighbors – see picture). Is it possible to write an integer in each square in such a way that the sum of each number with its 4 neighbors is equal to 13? If yes, show how. If no, explain why not.

5. Two girls are playing a game. The first player writes the letters A or B in a row, left to right, adding one letter on her turn. The second player switches any two letters after each move by the first player (the letters do not have to be adjacent), or does nothing, which also counts as a move. The game is over when each player has made 2011 moves. Can the second player plan her moves so that the resulting letters form a palindrome? (A palindrome is a sequence that reads the same forward and backwards, e.g. AABABAA.)