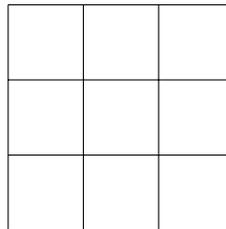


1. **The 15 game**

- (a) Play the game a few times with a partner. As you play, think about why you made certain choices.
- (b) What are all of the winning sets of cards? That is to say, what are all of the possible ways that three distinct whole numbers between 1 and 9 can add up to 15?

- (c) A *magic square* is a way of filling an $n \times n$ grid with the numbers $1, 2, 3, \dots, n^2$ in such a way that the sum of the numbers in each row, each column, and each diagonal is the same.



- (d) What is the magic sum in a 3×3 magic square?
- (e) Construct a 3×3 magic square.
- (f) What is the connection between the 15 game and the magic square you constructed?

Here are some more problems to think about.

2. A community of amoebas lives in a pond. Each day, each of the amoebas splits, and instead of one amoeba we now have two. The next day, each of those amoebas splits in two, and so on. Thirty days later, the pond is completely filled with amoebas! On which day was the pond halfway full of amoebas?
3. There are 100 politicians at a convention. Each politician is either crooked or honest. We know that:
 - (a) At least one of the politicians is honest.
 - (b) Given any two politicians, at least one of them is crooked.

How many politicians are honest, and how many are crooked? Explain your answer.

4. You have two sand clocks. One measures exactly 7 minutes, and the other measures exactly 11 minutes. It takes 15 minutes to boil an egg. How can you measure 15 minutes if you can only use the sand clocks?
5. At the North Cascades National Park, all trails either go uphill or downhill. A group of hikers can hike at a rate of 6km/hour if they are hiking downhill, but only 3km/hour if they are hiking uphill. They plan to hike from Cascade Pass to Stehekin and back, a hike that is 30km in each direction. How long will the hike take them?
6. Goldilocks enters the home of the three bears – Papa Bear, Mama Bear, and Baby Bear. Each bear is wearing a shirt of a different color – either red, green, or blue. All the bears look the same to Goldilocks, so she cannot otherwise tell them apart.

The bears in the red and blue shirts each make two statements, one of which is true and the other is false.

The bear in the red shirt says: “I am Blue’s dad. I am Green’s daughter.”

The bear in the blue shirt says: “Red and Green are of opposite genders. Red and Green are my parents.”
7. Which is larger, the sum of all of the even numbers from 0 to 100, or the sum of all of the odd numbers from 1 to 99? How much larger is it?