

Extra Problems

1 Bim always tells the truth, and Bom always lies. What question should you ask to each of them if you want to get the same answer? (Assume that you don't know who is who.)

2. **(Challenge Problem)** Johnny the Fifth Grader decided to make a Rubik's Cube. He took a $3 \times 3 \times 3$ wooden cube and painted each of the six faces in a different color. After that, he sawed this cube into 27 small $1 \times 1 \times 1$ cubes. (It took him 6 cuts total: two horizontal ones, two vertical ones parallel to the front face, and two vertical ones parallel to the side face.)

(a) How many of the small cubes have exactly three painted faces? Two painted faces? One painted face? How many cubes have no painted faces at all?

(b) Johnny's friend Sally says that Johnny should have stacked the parts of the cube together while sawing. This way, he would have achieved the same result with fewer than 6 cuts. Is Sally correct? Either show how to get 27 small cubes with fewer than 6 cuts, or explain why Sally is wrong.