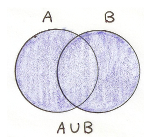
Sets 2

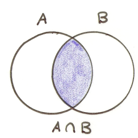
Montlake Math Circle

February 3, 2013

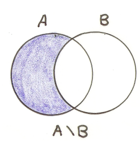
Union The union of two sets A and B, written $A \cup B$, is the combination of the two sets.



Intersection The intersection of two sets A and B, written $A \cap B$, is the overlap of the two sets.



Difference The difference of two sets A and B, written $A \setminus B$, is what is left of the first set after the elements of the second set are removed.



Empty set The empty set, written $\varnothing$, is the set containing no elements.

Problem 1 Let A, B, and C be sets. Determine which of the following statements are true using Venn diagrams:

$$(A \cap B) \cap C = A \cap (B \cap C)$$

$$(A \cup B) \cup C = A \cup (B \cup C)$$

$$(A \setminus B) \setminus C = A \setminus (B \setminus C)$$

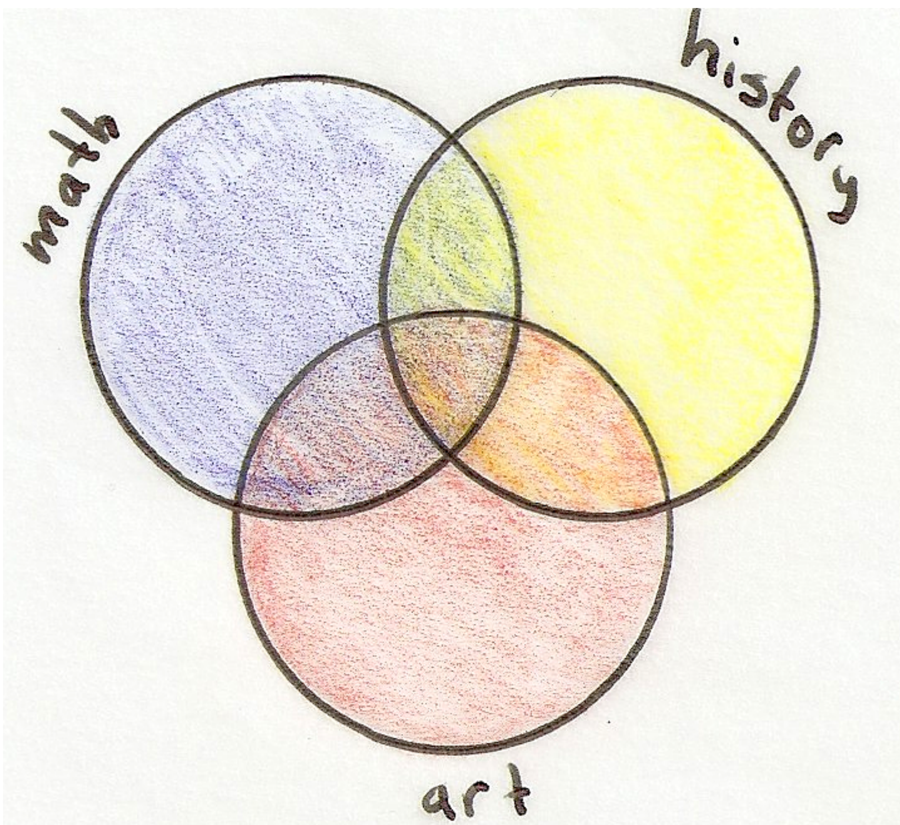
Problem 2 In a group of students there are 30 that take math, 20 that take art, and 21 that take history. 13 students take math and art, 12 take math and history, and 11 take art and history. You know there are 4 students that take all three classes.

Let M be the students that take math, A be the students that take art, and H be the students that take history.

Part 1 Represent the following groups of students in terms of the sets defined above:

Students that take math but not art:
Students that take art or history:
Students that take art and history:
Students that take only math:
Students that take math or art but not history:
Students that take math and art but not history:
Students that take all three:

Part 2 Label the disjoint (non-overlapping) regions of the Venn diagram in terms of the sets M, A, and H.



Part 3 Now using your diagram find how many students there are in total and how many students take exactly three, two, and one of the classes.

Definitions:

Union For all x, $x \in A \cup B$ if and only if $x \in A$ or $x \in B$.

Intersection For all x, $x \in A \cap B$ if and only if $x \in A$ and $x \in B$.

Difference For all x, $x \in A \setminus B$ if and only if $x \in A$ and $x \notin B$.

Empty set For all x, $x \notin \varnothing$

Problem 3 Prove the following statements:

Proposition 4: Let A be a set. Then $A \cap A = A$, $A \cup A = A$, and $A \setminus A = \varnothing$.

Proof of $A \cap A = A$: Let be a set.

Let $x \in A$. Then $x \in A$ $x \in A$ by definition of intersection. Therefore $x \in$ by logic. So $\subseteq$ by definition of subset.

Let $x \in A$. Therefore $x \in A$ $x \in A$ by logic. Therefore by definition of intersection. Therefore $\subseteq$ by definition of subset.

Therefore $=$ by Proposition .

Proof of $A \cup A = A$: Similar to above.

Proof of $A \setminus A = \varnothing$: Let be a set.

Let $x \in A \setminus A$. Then x x by definition of difference, which is a contradiction. Therefore by logic $x \in \varnothing$. Therefore $\subseteq$ by definition of subset.

Let $x \in \varnothing$. Therefore $x \in$ $\setminus$ by logic. Therefore $\subseteq$ by definition of subset.

Therefore $=$ by Proposition .

Proposition 5: Let A and B be sets. Then $A \cap B = B \cap A$ and $A \cup B = B \cup A$.

Proof of $A \cup B = B \cup A$: Let and be sets.

Let $x \in A \cup B$. Then $x \in$ $x \in$ by definition of union. Then $x \in$ or $x \in$ by logic. Therefore $x \in$ $\cup$ by definition of union. Therefore $\subseteq$ by definition of union.

Let $x \in B \cup A$. Then by definition of union. Then $x \in$ or $x \in$ by logic. Therefore by definition of union. Therefore $\subseteq$ by definition of subset.

Therefore $=$ by Proposition .

Proof of $A \cap B = B \cap A$: Similar to above.

Problem 5: Super Challenge Problem! Prove the following statements:

De Morgan's Laws: Let A, B, and C be sets. Then $C \setminus (A \cap B) = (C \setminus A) \cup (C \setminus B)$ and $C \setminus (A \cup B) = (C \setminus A) \cap (C \setminus B)$.

Hint: First draw Venn diagrams to think about why the laws are true!