We’ve seen that it’s impossible to walk the city of Konigsberg in the following way:

1. Start on some particular island.
2. Cross each bridge exactly once.
3. Return to the same island where you started.

This worksheet has some more problems about cities with islands and bridges. For the problems on this worksheet, don’t just answer “yes” or “no” – make sure you have a convincing argument for your answer. You don’t need to write out a full answer, but make sure you write enough that you remember your argument so we can talk about it when we meet. Feel free to draw pictures, too, if that helps.

One other note: we’re not interested in “trick” solutions to these problems. Sometimes people try to do things like cross a bridge half way, then double back, and other such complicated ways to get around the rules. These are not valid solutions. If you’re trying to do anything but simply crossing one bridge after another, start to finish, then the people of Konigsberg will tell you that you haven’t solved their problem!
Problem 1  Suppose the people of Konigsberg decide that, in light of our discovery that a true
Konigsberg walk is impossible, it would be okay if they relaxed the third condition. A “Konigsberg
Tour” is the same as a Konigsberg Walk, except you need not finish on the same island where you
started. Is it possible to do a Konigsberg Tour?
Problem 2. If you answered in problem 1 that a Konigsberg Tour is possible, draw the tour here (please be neat enough that you can be absolutely certain that no bridge is crossed twice). If you answered that it is not possible, then the people of Konigsberg have another question for you. They so badly would like to make a Konigsberg Tour of their city that they are willing to construct another bridge! Where should they construct it? Do they have multiple choices in the matter?
Problem 3  A key part of mathematics is generalization: taking ideas from a specific problem, like walking the city of Konigsberg, and making them more general. Let’s open up the discussion to any configuration of islands and bridges, which we will call a “city” (so when we talk about cities, they don’t have to be cities that exist in the real world – we are free to make up configurations of islands and bridges).

As before, a bridge is between exactly two islands, a pair of islands may be connected by any number of bridges (including 0), and the only way to get between islands is to cross bridges. Now, it doesn’t make much sense to talk about “Konigsberg Walks” and “Konigsberg Tours” any more, since we’re talking about cities other than Konigsberg. From now on, we’ll use their real mathematical names: “Eulerian Circuit” for a Konigsberg Walk and “Eulerian Tour” for a Konigsberg Tour (named for the great mathematician Euler, who originally solved these Konigsberg problems).

Konigsberg has four islands. Does there exist a city with five islands where an Eulerian Tour is possible? An Eulerian Circuit? Six islands?
Problem 4  Are there cities with five and six islands where an Eulerian Tour is not possible?
**Problem 5 (challenge)** You’ve had some practice now with cities that do and do not have Eulerian Tours. We saw in the case of Konigsberg that if a city has an island with an odd number of bridges then there is no Eulerian Circuit. This begs another question: if a city’s islands all have an even number of bridges touching them, does it have to have an Eulerian Circuit?