A Game: Pico, Fermi, Bagel

Today we will look at the logic that underlies the game "Pico, Fermi, Bagel".

How to Play

The game begins with one person secretly choosing a number with no repeated digits. We will start with a three-digit number. Then others attempt to guess the number, and the one who chose the number responds to each guess as follows:

- If the guess has no numbers correct, they respond: "Bagel."
- For each digit the guess has correct, but in the wrong place, they respond: "Pico."
- For each digit the guess has correct and in the correct place, they respond: "Fermi."

As an example, let's say that you thought of the secret number 489.

Guess 1: 362 Bagel — no digit is correct.

Guess 2: 820 Pico — the 8 is in the wrong place.

Guess 3: 418 Pico Fermi — the 8 is in the wrong place, the 4 is in the correct place.

Guess 4: 518 Pico — the 8 is in the wrong place.

Guess 5: 487 Fermi Fermi — the 4 and 8 are in the correct place.

Guess 6: 489 Fermi Fermi Fermi — all digits are in the correct place.

We will play a few rounds of the game together, and then try to solve a couple problems related to the game.
Problem 1. Given the following four guesses, it is possible, without any further guesses, to determine the secret number?

Guess 1: 6152  Pico Fermi
Guess 2: 4182  Pico Pico
Guess 3: 5314  Pico Pico
Guess 4: 5789  Fermi

Problem 2. What is the minimum number of guesses needed that will allow you to always determine a secret number:

Where the secret number has one digit?

Where the secret number has two digits?

Where the secret number has three digits?