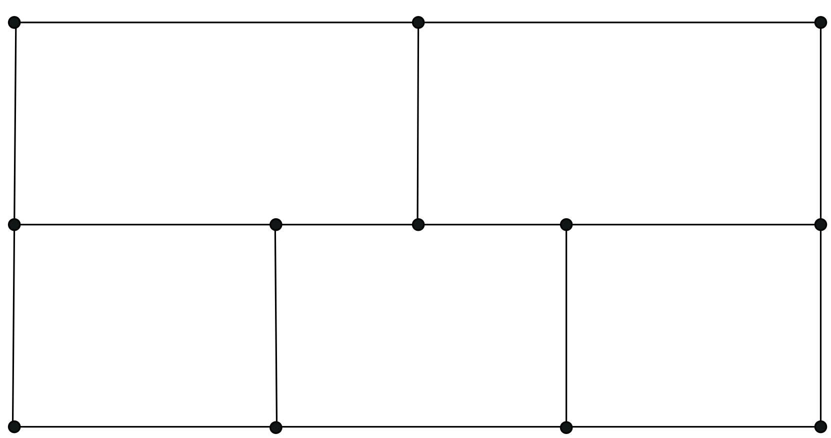
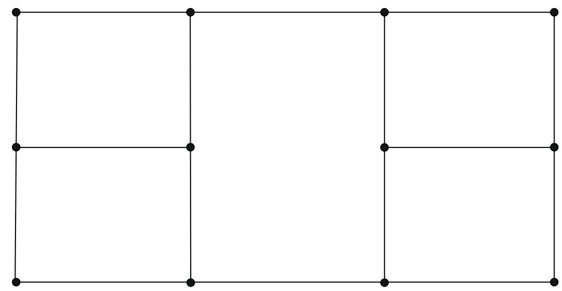
Graphs and Related Puzzles

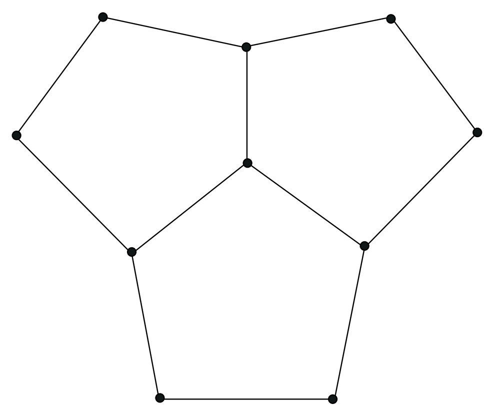
Problem 1 (*Graphs*)**.** In the distant future, regular travel between planets will become possible. Suppose spacecraft travel along the following routes: Earth–Mercury, Pluto–Venus, Earth–Pluto, Pluto–Mercury, Mercury–Venus, Uranus–Neptune, Neptune–Saturn, Saturn–Jupiter, Jupiter–Mars, and Mars–Uranus. Can a traveler get from Earth to Mars?

Problem 2. Can you place a shoe lace on this figure so that it crosses each of the 16 line segments exactly once? If so, demonstrate. Otherwise, justify why not.

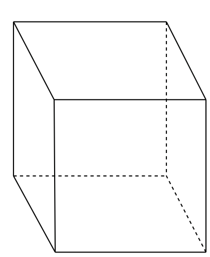


Problem 3. Can you place a shoe lace on either of these figure so that it crosses each of the their line segments exactly once?





Problem 4. Suppose I unfold a paperclip and find that it is 6 inches. long. I want to fold it into a wire-frame cube that is 1/2 inch by 1/2 inch buy 1/2 inch. Can this be done without cutting the paperclip into smaller pieces? If not, how many cuts do I have to make to be able to form my cube?



Problem 5. Is it possible in any of the following graphs to start at one vertex and travel across each edge exactly once arriving at the vertex you started from? Such a trip is called an Eulerian cycle. Is there a characteristic that distinguishes the graphs that have Eulerian cycles from those that don't?

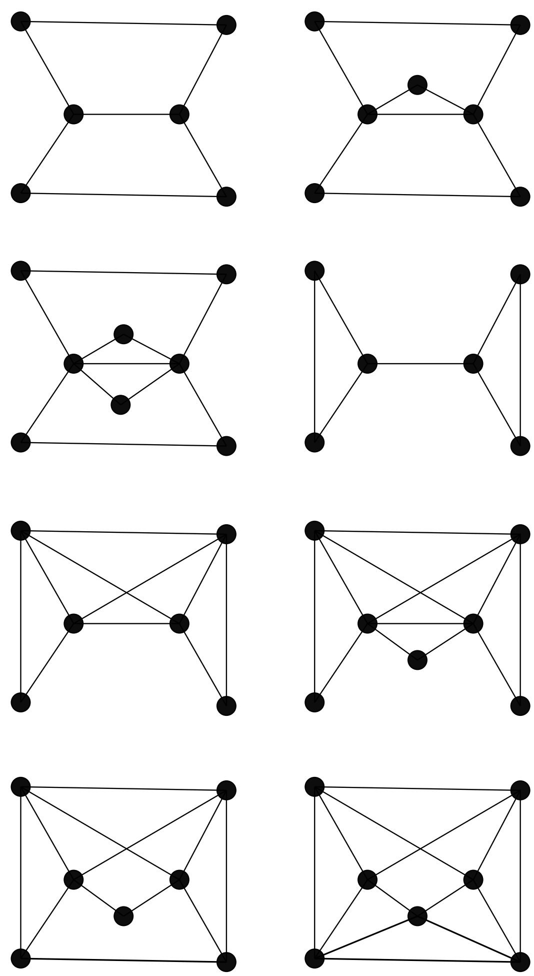


Figure 1: Graphs that may or may not have Eulerian cycles.

Problem 6. In a certain kingdom, the are 6 cities, and three roads lead out of each city. How many roads are there in the kingdom? Is there an Eulerian cycle between the cities? What if there were 6 cities, and two roads lead out of each city: How many roads? Is there an Eulerian cycle?