



## Lesson #41: Exploding Dots

Approximate lesson length: 1 day

Following lesson # \_\_ (lesson 9 binary)

Precedes lesson # \_\_ (lesson 11 jeopardy)

Fits well with lesson # \_\_ (other different base ones)

### Lesson Objectives:

- Familiarize the students with the idea of other bases (not 10 or 2)
- Explain generalized addition in different bases

### Materials:

- Instructor's White board / Chalkboard (if not available, try taping blank sheets of paper to a wall)
- Paper and pencils for the students

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Now that we've covered binary (base 2) in the previous lesson, the concepts can easily be extrapolated to any base you want. To do this for some base  $x$ , just use the same chart idea with columns that are powers of  $x$  that you used in the previous lesson. Here's the generalized chart:

column	$x^2$	$x^1$	$x^0 = 1$
number			
column value (base 10)			

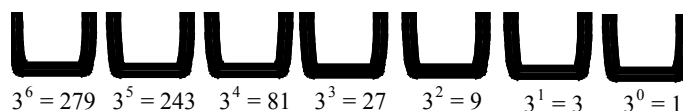
Addition still works the same way too, although it can be a bit tougher to understand for bigger bases. This lesson uses an abstraction to take the numbers and the symbols out of the way and get to the heart of how addition works for any base you want.

### Part 1: Loading up the containers

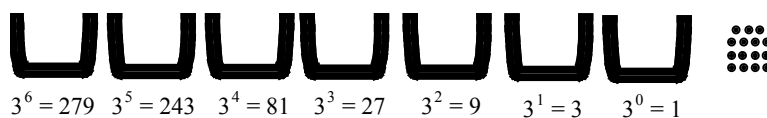
Start by drawing a series of containers on the board that look like this (the number isn't important, just draw a bunch of them):



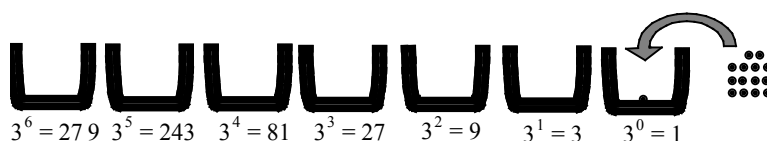
These containers are going to represent the place columns for your number (from right to left:  $x^0$ ,  $x^1$ ,  $x^2$ , ...). You probably want to start off with a decently low base number, so have the students give you numbers and choose a low one to be the base. For this example we'll use base 3. Label the containers with their appropriate column labels like this:



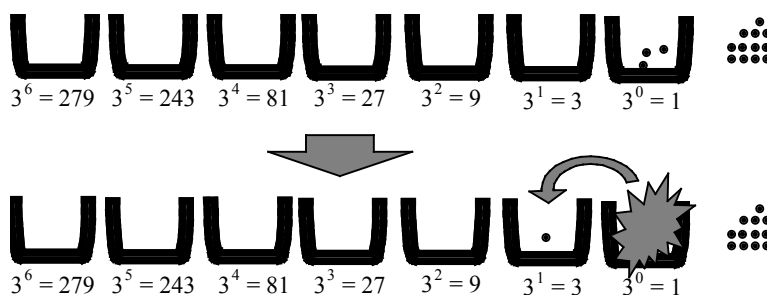
Next, get a number from the students (in base 10) that isn't too large. For this example, let's use 15. Make a pile of 15 dots to the right of your containers.



Now, one by one add them to the right most container.



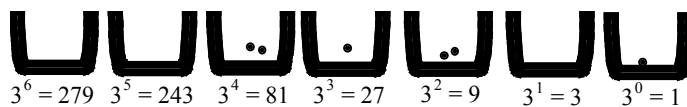
When the right most container has the magic number of dots in it (in this case 3), it explodes and all the dots disappear except for one that flies into the next container to the left.



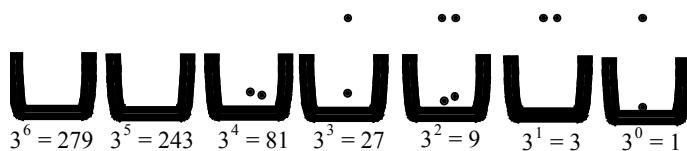
Continue this process until you have used up all the dots in the pile. Every time a container has the magic number of dots in it, it explodes and one dot flies off to the next container to the left. This can sometimes cause a chain reaction. If there are two dots each in the first two containers and you add one more, the first container will explode making the second one have 3 which causes it to explode. The end result would be one dot in the third container and none in either the first or second. After walking through one example, the students should have a pretty good idea how it works to convert a number from base 10 (the number you started with) to whatever your new base is. If you think you'll have time, or the students still seem confused, it can't hurt to do another example with a different base.

## Step 2: Addition

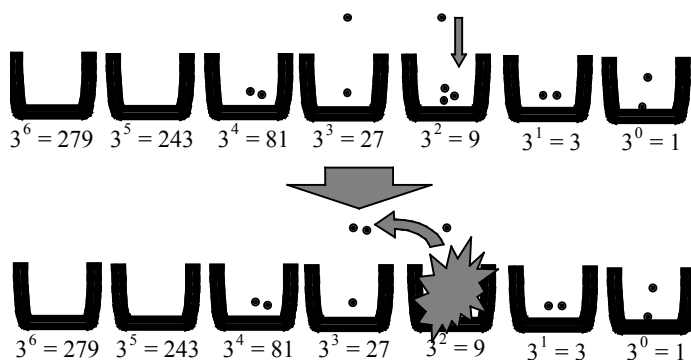
Now that the students have figured out how to represent numbers with the dots and containers, you shouldn't have to convert every number from base 10. Get a new base to work with from the students and have them give you a number in that base. Remember that a number in base  $x$  can have any digit whose value is less than  $x$ . For example if you are using base 6, a number can only have 0, 1, 2, 3, 4, 5 for digits. Similarly, if you are using base 16, a number can have 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 for digits and then you can make up symbols to represent digits equal to 10, 11, 12, 13, 14, 15. A common practice here is to assign letters for the higher digits (a, b, c, etc...). Let's keep using base 3 for the example, and let's use 21201 for our example number. Fill the appropriate containers up with the right number of dots.



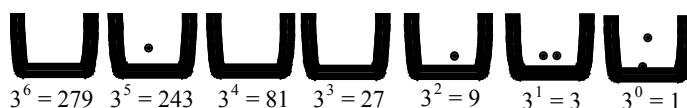
Next, get a second number, lets use 1221, and place its dots above the appropriate containers.



Following the same rules as before, whenever a container has the magic number of dots in it it explodes and one dot flies to the left. The only catch here is that even once a container explodes, if there are still more dots to add to that container, you have to add the rest of them before you start adding any to the next one. The dot that flies to the left gets added to the pile of dots waiting to get added but it doesn't actually get added until after all the dots are finished being added to the current container.



Finish adding all of the dots that are hanging above the containers to finish the addition. The result of this example looks like this:



Do a few examples of addition and try using different bases. Don't worry if the students don't have it down for all bases right away. The next step gives you time to talk to them individually.

### Step 3: Challenge Problems

This step switches back to just using numbers. Give each student a few addition problems in different bases (start out with the same bases that you've done examples with). Let each student go to work trying to solve the problems. Walk around and watch for trouble spots. Take this opportunity to help clarify the ideas to the student on an individual basis. This can prove to be the most useful step for getting the students to fully understand the idea behind addition in different bases.

Note: This lesson was adapted from one taught by Robert and Ellen Kaplan