Big Sums

Problem 1. What is the sum of the even numbers from two to $2n$? If you find an answer, can you explain why it works?

$$
\begin{aligned}
n = 1 : \quad & 2 \\
n = 2 : \quad & 2 + 4 = \\
n = 3 : \quad & 2 + 4 + 6 = \\
n = 4 : \quad & 2 + 4 + 6 + 8 = \\
n = 5 : \quad & 2 + 4 + 6 + 8 + 10 = \\
n = 6 : \quad & 2 + 4 + 6 + 8 + 10 + 12 = \\
& \quad \vdots \\
n = n : \quad & 2 + 4 + 6 + \cdots + 2(n - 1) + 2n =
\end{aligned}
$$

Problem 2. What is the sum of the multiples of three numbers from three to $3n$?

Problem 3. What is the sum of the odd numbers from one to $2n - 1$? If you find an answer, can you explain why it works?

$$
\begin{aligned}
n = 1 : \quad & 1 \\
n = 2 : \quad & 1 + 3 = \\
n = 3 : \quad & 1 + 3 + 5 = \\
n = 4 : \quad & 1 + 3 + 5 + 7 = \\
n = 5 : \quad & 1 + 3 + 5 + 7 + 9 = \\
n = 6 : \quad & 1 + 3 + 5 + 7 + 9 + 11 = \\
& \quad \vdots \\
n = n : \quad & 1 + 3 + 5 + \cdots + [2(n - 1) - 1] + [2n - 1] =
\end{aligned}
$$

Problem 4. What is the sum of the powers of 2 from one to 2^n? If you find an answer, can you explain why it works?

$$2^0 = 1: \qquad 1$$
$$2^1 = 2: \qquad 1+ \quad 2 =$$
$$2^2 = 4: \qquad 1+ \quad 2+ \quad 4 =$$
$$2^3 = 8: \qquad 1+ \quad 2+ \quad 4+ \quad 8 =$$
$$2^4 = 16: \qquad 1+ \quad 2+ \quad 4+ \quad 8+ \quad 16 =$$
$$2^5 = 32: \qquad 1+ \quad 2+ \quad 4+ \quad 8+ \quad 16+ \quad 32 =$$
$$2^6 = 64: \qquad 1+ \quad 2+ \quad 4+ \quad 8+ \quad 16+ \quad 32+ \quad 64 =$$
$$\vdots$$
$$2^n: \qquad 1+2+4+\cdots+2^{n-1}+2^n =$$

Problem 5. What is the sum of the powers of 3 from one to 3^n? If you find an answer, can you explain why it works?

$$3^0 = 1: \qquad 1$$
$$3^1 = 3: \qquad 1+ \quad 3 =$$
$$3^2 = 9: \qquad 1+ \quad 3+ \quad 9 =$$
$$3^3 = 27: \qquad 1+ \quad 3+ \quad 9+ \quad 27 =$$
$$3^4 = \quad: \qquad 1+ \quad 3+ \quad 9+ \quad 27+ \quad =$$
$$3^5 = \quad: \qquad 1+ \quad 3+ \quad 9+ \quad 27+ \quad + \quad =$$
$$3^6 = \quad: \qquad 1+ \quad 3+ \quad 9+ \quad 27+ \quad + \quad + \quad =$$
$$\vdots$$
$$3^n: \qquad 1+3+9+\cdots+3^{n-1}+3^n =$$

Problem 6. What is the sum of the powers of 4 from one to 4^n? If you find an answer, can you explain why it works?

$$4^0 = 1: \qquad 1$$
$$4^1 = 3: \qquad 1+ \quad 4 =$$
$$4^2 = 9: \qquad 1+ \quad 4+ \quad 16 =$$
$$4^3 = \qquad : \qquad 1+ \quad 4+ \quad 16+ \qquad =$$
$$4^4 = \qquad : \qquad 1+ \quad 4+ \quad 16+ \qquad + \qquad =$$
$$4^5 = \qquad : \qquad 1+ \quad 4+ \quad 16+ \qquad + \qquad + \qquad =$$
$$4^6 = \qquad : \qquad 1+ \quad 4+ \quad 16+ \qquad + \qquad + \qquad + \qquad =$$

$$\vdots$$

$$4^n: \qquad 1+4+16+\cdots+4^{n-1}+4^n =$$

Problem 7. Can you guess the sum of powers of five from one to 5^n?

Problem 8. Look at the following triangle of numbers. Do you see the pattern? Can you continue the triangle? Do you see anything related to our previous summation problems? What are the sums of the rows?

```
                1
             1     1
          1     2     1
       1     3     3     1
    1     4     6     4     1
 1     5    10    10     5     1
1    6    15    20    15     6    1
1  7   21   35   35   21   7   1
```