A Selection of Problems We've Done in Mountlake Math Challenge 2009-2010

Nim

NIM is a game played with piles of stones (or pennies). You will need a partner to play this game. Here are the rules:

1. On your turn, you can take as many stones you want from one pile.
2. You must take at least one stone on each turn.
3. Players alternate taking turns.
4. The player who takes the last stone from the last pile wins.

Try playing NIM with your partner. Keep track of who goes first in each game. There is always one player who is guaranteed to win if he/she plays according to a certain strategy. It is your job to figure out what this strategy is. Decide if Player 1 wins or Player 2 wins in each game.

Game 1: Pile 1 has two stones. Pile 2 has four stones. Pile 3 has six stones.
Winner is: Player 1 or Player 2
Strategy:

Game 2: Pile 1 has three stones. Pile 2 has four stones. Pile 3 has eight stones.
Winner is: Player 1 or Player 2
Strategy:

Game 3: Pile 1 has one stone. Pile 2 has four stones. Pile 3 has six stones.
Winner is: Player 1 or Player 2
Strategy:

The Princess or the Tiger

from: Raymond Smullyan

The King of Indrahabad had heard of a story where a prisoner had to choose between two doors: behind one there was a princess, behind the other a tiger. If the prisoner chose the princess, he could marry her; if he chose the tiger, he would probably be eaten.

The King thought this was a nice idea for his prisoners. But he didn't want to leave things completely to chance, so he decided that he would put some signs up on the doors. The signs would give some clues as to what was behind the doors. A clever prisoner who can reason logically would be able to save his own life. And, of course, in that way the King could ensure that the husbands for all his daughters would be smart!

To make it a bit more difficult the King decided that it would also be possible that there are princesses behind both doors or tigers behind both doors. But there will never be more than one princess per room or one tiger per room.

The following two signs were put up on the doors:

I
In one of these rooms, there is a princess and, in the other, there is a tiger.

II
In this room, there is a princess and, in the other room, there is a tiger.

The prisoner asked: "Is it true, what the signs say?"
The King said: "One is true and one is false".
Which door would you take if you were the prisoner? Why?

Problem 1 (*Another logic puzzle*)**.** Suppose that you are a prisoner, and you are confronted with two doors: one leading to freedom, and one leading to the executioner's chamber, but you don't know which is which. A sentry guards each door. You know that one sentry always lies, and one sentry always tells the truth. Again, you don't know which is which. You are allowed to ask one of the sentries one question. What is a question you ask a sentry that will tell you with certainty which door leads to freedom?

Graphs

Problem 2. How many times can three people shake hands if each person shakes hands with each other person exactly once? Same question for four people? Five people? Six? Ten?

Problem 3. In a certain kingdom, the are 8 cities, and three roads lead out of each city. How many roads are there in the kingdom? What if there were 12 cities, and three roads leading out of each? Now suppose there are 20 cities and four roads out of each? Lastly, what if there are three cities with three roads leading out of each?

The Pigeon Hole Principle

Problem 4. A drawer contains socks of two different colors: black or white. If you pull socks out of the drawer without looking, what is the minimum number of socks to pull out to assure yourself of a match?

Problem 5. The population of King county is about $1,800,000$ people. If humans have no more than one million hairs on their heads, show that there must be two people in the county with the same number of hairs on their heads.

Problem 6. Thirteen crates of apples arrives at a warehouse. Each crate contains apples of one of three varieties: Delicious, Granny Smith, or Pink Lady. Are there four crates of one variety? Why?

Problem 7. Are there five crates of one variety? Why?

Problem 8. Show that if fifteen people attend a party and some shake hands with others (but not with themselves), then at the end, there are at least two people who have shaken hands with the same number of people.

How Do We Do That?

Problem 9. There are two egg timers: one for 7 minutes and one for 11 minutes. How can we use these timers to measure exactly 15 minutes?

Problem 10. Find a two digit number, the sum of whose digits does not change when multiplied by a one digit number.

Parity or Paired Things

Problem 11. Seven gears are arranged on a plane as shown below. Can all the gears rotate simultaneously?

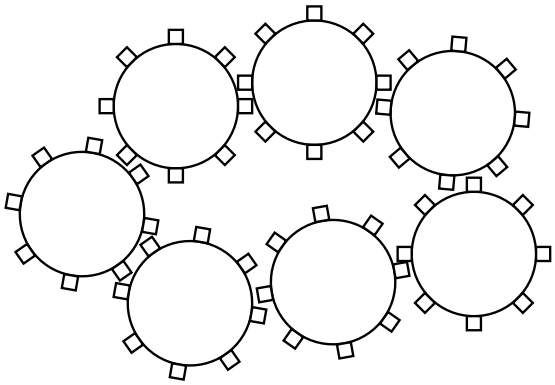

Figure 1:

Problem 12. Remembering that a knight's move in chess is straight two squares and over one, show that if a knight starts in one corner of a chessboard and after several moves returns to it, then it must have made an even number of moves.

Problem 13. Can a 5×5 square checkerboard be covered by 1×2 dominoes? How or why not?

Divisibility by 2, 3, 5, 9, and 11

Problem 14. How can you quickly determine whether a number is divisible by 2 or by 5? More importantly, why does this work?

Problem 15. How can you quickly determine whether a number is divisible by 3? And, this is harder, why does this work? Is there a similar approach to divisibility by 9?

Problem 16. How can you quickly determine whether a number is divisible by 11?

Lines and Squares

Problem 17. In each of the following figures, find the total number of intersections made by the dotted lines. Then find the total number of rectangles formed by the dotted lines. Do you see any pattern to the progression? Do you see a relationship with the line problem?

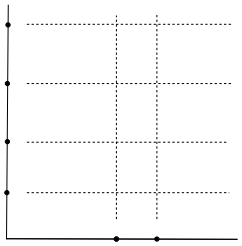

Figure 2: Number intersections : Number Rectangles:

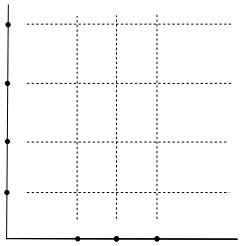

Figure 3: Number intersections : Number Rectangles:

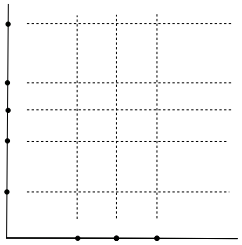

Figure 4: Number intersections : Number Rectangles:

Permutations

Problem 18. How many rearrangements of letters of the word "SPECIAL" are there?

Problem 19. Which word has more arrangements of letters: "MEAT" or "MEET"? Why? How many arrangements does each word have? How many arrangements does "MEEM" and "MEEE" have?

"MEAT":

"MEET":

"MEEM":

"MEEE":

Problem 20. How many arrangements are there of the letters in "CARAVAN"?

Problem 21. How many ways are there to put n *different* balls into m different boxes?

Problem 22. How many ways are there to put n *identical* balls into m different boxes?