Graphs (again)

Problem 1 (*Think about using graphs*)**.** A chessboard has the form of a cross, created from a 4×4 chessboard by deleting the corner squares. Can a knight travel around this chessboard, using the usual knight's move, passing through each square exactly once, and end up on the same square it starts on?

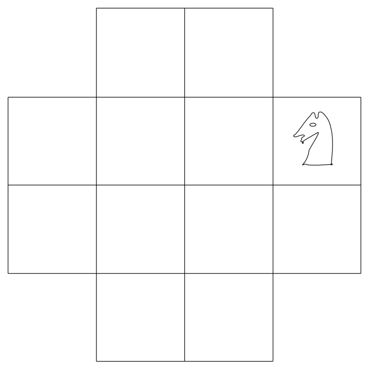

Figure 1: Knight's tour.

Problem 2 (*Again, think about using graphs*)**.** Four knights are positioned on a 3×3 chessboard as shown on the first chessboard below. Can they move to the positions shown on the second chessboard?

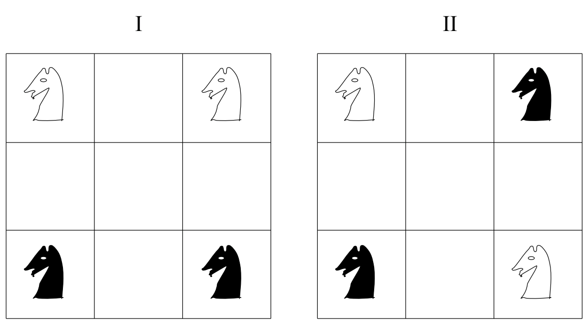

Figure 2: Knight's positions, before and after.

Problem 3. Is it possible in any of the following graphs to start at one vertex and travel across each edge exactly once arriving at the vertex you started from? Such a trip is called an Eulerian cycle. Is there a characteristic that distinguishes the graphs that have Eulerian cycles from those that don't?

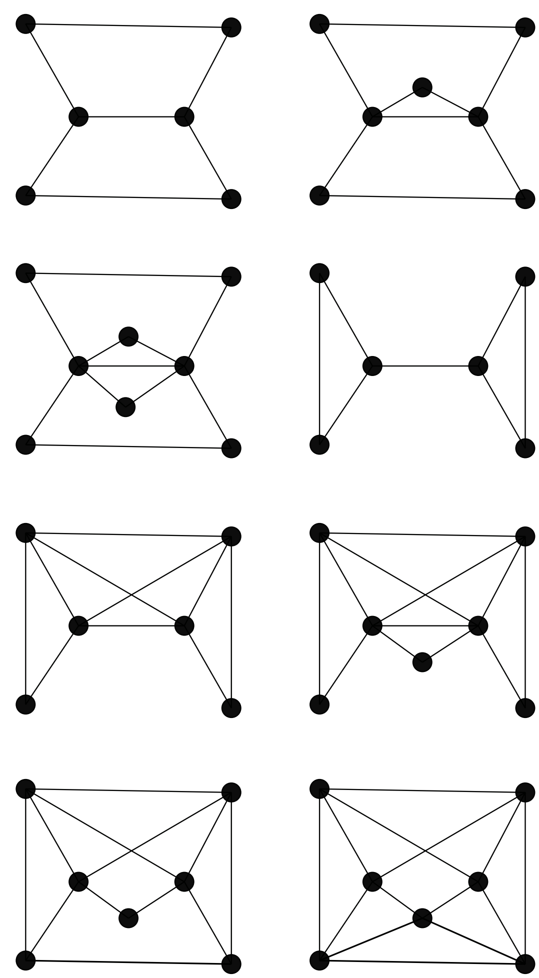

Figure 3: Graphs that may or may not have Eulerian cycles.

Problem 4. Prove that the sum of the degrees of the vertices of any finite graph is even.

Problem 5. Show that if n people attend a party and some shake hands with others (but not with them-selves), then at the end, there are at least two people who have shaken hands with the same number of people.