Geometry Puzzles and Graphs

Problem 1. Toothpicks are arranged as shown below. Move two toothpicks to form four squares each with side length equal to one toothpick.

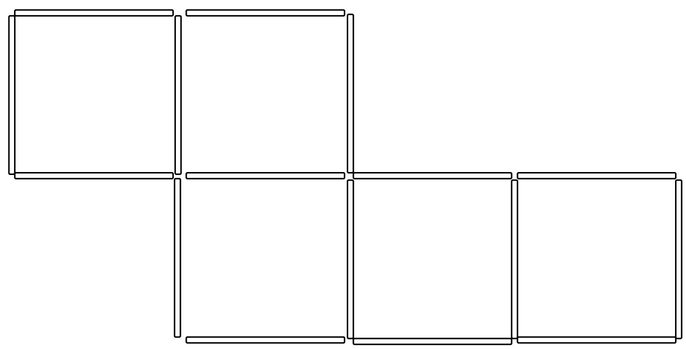

Figure 1: Toothpicks, move two.

Problem 2. Toothpicks are arranged as shown below. Remove four toothpicks to leave two equilateral triangles. Remove three toothpicks to leave, again, two equilateral triangles. Finally, remove just two toothpicks to leave two equilateral triangles. *Loose ends* are not allowed.

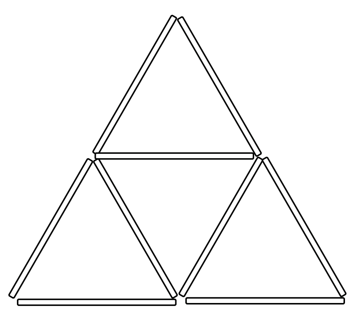

Figure 2: Toothpicks, remove four, three or two, leaving two equilateral triangles.

Problem 3. Is it possible to arrange six pencils so that each pencil touches each of the others? If so, how?

Problem 4. Ten pennies are arranged as shown below. What is the minimum number of pennies we must remove so that no three of the remaining pennies lie on the vertices of an equilateral triangle.

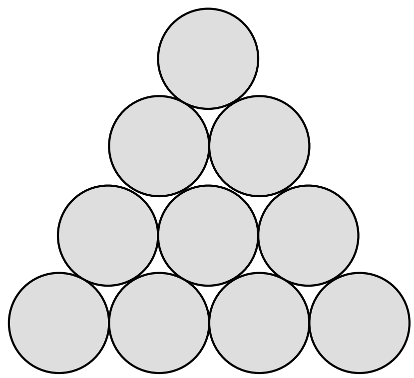

Figure 3: Pennies.

Problem 5. Draw six lines segments to form eight triangles. See if you can find two different ways of accomplishing this.

Problem 6 (*Graphs*)**.** In the distant future, regular travel between planets will become possible. Suppose spacecraft travel along the following routes: Earth–Mercury, Pluto–Venus, Earth–Pluto, Pluto–Mercury, Mercury–Venus, Uranus–Neptune, Neptune–Saturn, Saturn–Jupiter, Jupiter–Mars, and Mars–Uranus. Can a traveler get from Earth to Mars?

Problem 7. How many times can three people shake hands if each person shakes hands with each other person exactly once? Same question for four people? Five people? Six? Ten?

Problem 8. In a certain kingdom, the are 8 cities, and three roads lead out of each city. How many roads are there in the kingdom? What if there were 12 cities, and three roads leading out of each? Now suppose there are 20 cities and four roads out of each? Lastly, what if there are three cities with three roads leading out of each?

Problem 9. Can a kingdom in which 3 roads lead out of each city have exactly five roads? Can it have exactly six roads? Ten? Fifteen?

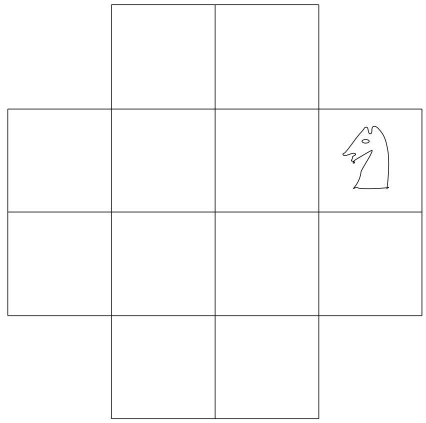

Figure 4: Knight's tour.

Problem 10 (*Think about using graphs*)**.** A chessboard has the form of a cross, created from a 4×4 chessboard by deleting the corner squares. Can a knight travel around this chessboard, using the usual knight's move, passing through each square exactly once, and end up on the same square it starts on?

Problem 11 (*Again, think about using graphs*)**.** Four knights are positioned on a 3×3 chessboard as shown on the first chessboard below. Can they move to the positions shown on the second chessboard?

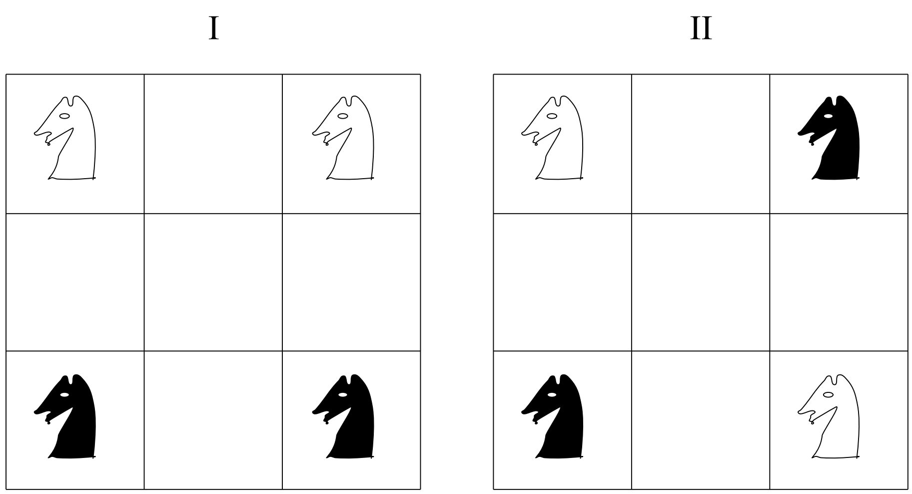

Figure 5: Knight's positions, before and after.