Montlake Math Challenge

Math Auction

Today we will be having a math auction. Here are the rules:

1. We will split into teams of 3 people. Each team will be given 500 Steve&Sam Math Fun Bucks. Each team will also need a name.
2. A problem will be given to all the groups. The problem will have many possible solutions, but one solution can be better than another.
    a) A sample problem is: What is the largest number of oranges that can fit into a cubical box that is 1 foot by 1 foot by 1 foot. Each orange is 4 inches in diameter.
    b) Team A might have an answer of 15 oranges, and Team B might answer 20 oranges. In this case, Team B has the better answer. Team A will have an opportunity to improve on its answer later.
3. Now for the auction part! Individually, the teams will try to solve the given problem, coming up with the best solution they can. A problem will have a certain opening cost. Say, for example, the orange problem has an opening cost of 5 Fun Bucks.
    a) When a team bids on a problem, they are betting that their solution is best. (So it's important to be secretive about your solutions!)
    b) Teams will all have a chance to increase the bid on a given problem. Here is an example:
        1. Opening bid: 5 Fun Bucks
        2. Team A bids 5 Fun Bucks
        3. Team B bids 10 Fun Bucks
        4. Team C bids 12 Fun Bucks
        5. Team A bids 15 Fun Bucks
        6. Team C bids 17 Fun Bucks
        7. Team B bids 20 Fun Bucks
        8. no one else wants to bid
    c) The team that wins the bid gives the amount of money they bid to Auctioneer Sam (in this case 20 fun bucks) and presents their solution to the class.
4. Now the problem is auctioned again. The team that wins the bid this time has to present a better solution than the one that was just presented. Because of this, it is important to have several solutions to the same problem.
5. This rebidding process continues until the groups can no longer come up with a better solution to the problem.
6. The team with the best solution wins all the bid money that Auctioneer Sam has collected.
7. There are 5 total problems. Each problem will be auctioned in this fashion. The team with the most money at the end wins!

Problem 1: Express the number 100 using only the digit 8 and the operations +, −, ×, and ÷. You can also use parentheses and exponents. Use the smallest number of 8's as you can.

Problem 2: A wooden stick is to be made into a ruler by marking it with labels. Your goal is to be able to measure any integer distance between 1cm and 20 cm (including 1cm and 20cm) without moving the ruler. That is to say, for any number $1 \leq N \leq 20$, there should be two tick marks on the ruler that are Ncm apart. What is the smallest number of tick marks you can use?

Problem 3: You are given a piece of paper in the shape of a circle. You can make 7 straight cuts through the circle without moving any pieces between cuts. What is the largest number of triangles you can make? (Triangles have 3 straight sides)

Problem 4: What is the largest number of rooks you can place on a chessboard so that each rook attacks an ODD number of other rooks.

Problem 5: What is the largest number of pieces shown below that can be placed on a 10 x 10 checkerboard so that none of them overlap?