

Montlake Math Challenge

October 16, 2008

Counting Problems

Problem 1: Three friends – Alice, Bob, and Carl – are standing in a circle. Each person shakes hand with everyone else in the circle.

a) How many hands does each person shake?

b) What is the total number of handshakes?

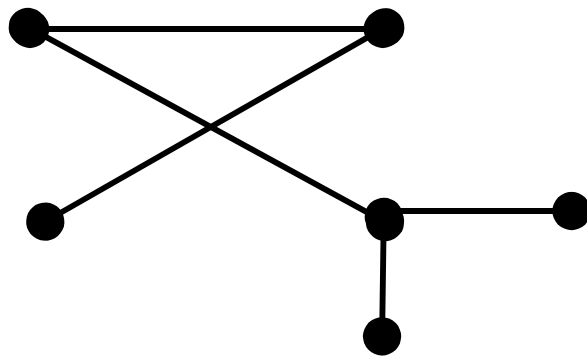
Problem 2: Five friends – Alice, Bob, Carl, Dominick, and Erica – are standing in a circle. Each person shakes hands with everyone else in the circle.

a) How many hands does each person shake?

b) What is the total number of handshakes?

Problem 3: In problems 1 and 2, multiply the number of people in the circle by the number of hands that each person shakes. How does this number compare to the number of total handshakes?

A **graph** is a collection of dots (called **nodes**) and lines (called **edges**) that connect the dots. This is an example of a graph:



This is an example of a graph with 6 nodes and 5 edges.

Problem 4: Draw a graph that represents the handshakes in problems 1 and 2. Draw a node corresponding to each person, and draw a line between each pair of friends who shake hands.

Problem 5: Draw a graph with 6 nodes and some edges (as many as you want). Label the nodes 1,2,3,4,5,6.

- a) How many edges touch node 1?
- b) How many edges touch node 2?
- c) How many edges touch node 3?
- d) How many edges touch node 4?
- e) How many edges touch node 5?
- f) How many edges touch node 6?
- g) Add up the numbers you got in parts (a)-(f).
- h) How many edges are there in your graph?
- i) How are the numbers in parts (g) and (h) related?