Your Name:_____

Montlake Math Challenge

March 12, 2009

Fibonacci Numbers

Introduction: Today we will be working with a special sequence of numbers called the Fibonacci sequence. To form the Fibonacci sequence, we start with two numbers, and to add another number to the sequence, we add the two numbers that come in line before it.

The first two numbers in the Fibonacci sequence are

1 1

the third number in the Fibonacci sequence is the sum of the first two

1 1 2

the fourth number in the Fibonacci sequence is the sum of the second and the third

1 1 2 3

Exercise 1: Write down the first twelve numbers in the Fibonacci sequence.

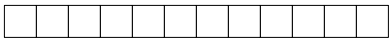
1 1 2 3 _____ _____ _____ _____ _____ _____ _____ _____

There is no reason that the first two numbers in our sequence have to be 1 1. Let's make a new sequence called the **Lucas sequence** in the same way as the Fibonacci sequence, but now we will make the first two numbers 2 1.

Exercise 2: Write out the first 12 numbers in the Lucas sequence:

2 1 3 4 _____ _____ _____ _____ _____ _____ _____ _____

Now let's try to answer a new question: Pretend I have a "checkerboard" that has one row and twelve columns:



I want to cover this checkerboard with the square dominoes and rectangular dominoes drawn below:

Question 3: In how many different ways can I cover the 1 × 12 board with these types of dominoes?

This question may seem really hard at first. Let's start with a smaller example. Pretend instead that I had a board that was four squares across. There are five ways to cover the board with square and rectangular dominoes: (I will color the square dominoes white and the rectangular dominoes black)

Question 3a: In how many different ways can I cover a single square (a 1 × 1 checkerboard) with these square and rectangular dominoes?

Question 3b: In how many different ways can I cover a 1 × 2 checkerboard with these square and rectangular dominoes?

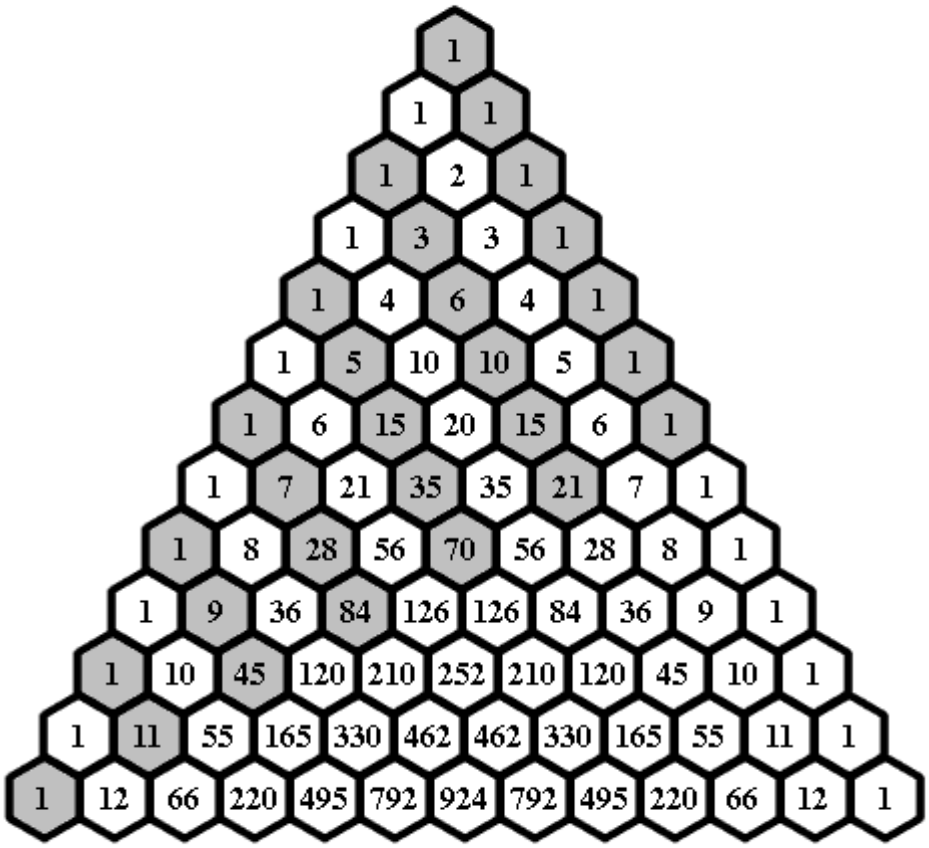
Question 3c: In how many different ways can I cover a 1 × 3 checkerboard with these square and rectangular dominoes?

Question 3d: Can you guess how many ways there are to cover a 1 × 5 checkerboard with these square and rectangular dominoes? How about a 1 × 6 checkerboard? How about a 1 × 12 checkerboard?

Question 3e: Why are your guesses in 7d correct? (Hint: Look at the pictures of all the different ways to cover a 1 × 4 board. Group these coverings into two different sets based on what kind of domino covers the square all the way to the right of each board.)

Fibonacci Numbers and Pascal's Triangle

In the picture of Pascal's triangle shown below, there are diagonals that move from the **lower left** corner of the triangle to the **upper right** side of the triangle. The diagonals alternate in colors. The first diagonal consists of the single grey 1 at the top of the triangle. The second diagonal consists of the single white 1 in the second row of the triangle. The third diagonal consists of two grey 1's – the first one in row 3 and the second one in row 2.



What is the sum of the numbers in each of the diagonals shown above?

Diagonal	Sum
1	
2	
3	
4	
5	
6	
7	