Montlake Math Challenge
March 6, 2007

Remember that i is the special number $\sqrt{-1}$, so that $i \times i = -1$. If we have $z = a + bi$, where a and b are ordinary real numbers, then we call the absolute value of z the value $|z| = \sqrt{a^2 + b^2}$. For example, if $z = i + 2$, then the absolute value of z is $\sqrt{1^2 + 2^2} = \sqrt{1 + 4} = \sqrt{5}$

1. On an attached sheet of graph paper or below, draw the real line from -10 to 10. Then draw the imaginary axis from $-10i$ to $10i$. Now draw a circle with radius 5 and center 0. (It should go through all of the points 5, -5, $5i$ and $-5i$). It turns out that these are the points with absolute value 5.

   How many other (whole number) complex points lie on this circle? Show that the points you found above are exactly on this circle by calculating their absolute values.

2. Find a complex number $z = a + bi$ that solves the equation.

$$3i + z = 15i$$

$$z + 2i = 1 + 2i$$

$$(2i + 3) + z = 5i + 2$$

$$z + (i - 1) = 2i + 3$$

3. (Harder) Find a complex number $z = a + bi$ that solves the equation.

$$3i \times z = 15i$$

$$z \times 2i = -14$$

$$(2i + 3) \times z = -4 + 6i$$

$$z \times (3i - 1) = 8i - 14$$