Montlake Math Challenge
Homework assigned 6 March 2007

We have the notation $X \bmod N$ which means divide X by N and take the remainder. For example, $2 \bmod 5 = 2$ ($2 \div 5 = 0$ remainder 2) and $19 \bmod 5 = 4$ ($19 \div 5 = 3$ remainder 4).

1. Find a pattern to complete the table below

n	$n \bmod 6$	$3^n \bmod 7$
1	1	3
2	2	2
3		
4		
5		
6	0	1
7	1	
8		
9		
10		
100		
1000		
1000000		

2. Using the same ideas, calculate $5^{1000} \bmod 7$.

One can use mod (officially, this is called "modular arithmatic") to quickly calculate the winner of a round of "Eany, meany, miney, moe, ..." Specifically, if your rhyme has X syllables and there are N people, you will finish on person $X \bmod N$.

3. Suppose your rhyme has 24 syllables. How many people can there be in the group so that it ends on the same person it started with? List all possible answers (including justification that you found all of them).

4. Suppose you want to rig the selection, so you try and come up with a rhyme that has the right number of syllables. For each of the following, figure out the smallest number of syllables you need. (For fun, you can come up with a rhyme too.)

   - You want to end on the second person of 5. To make things less obvious, you must go around at least twice.
   - You again want to end on the second person, but you *don't know* if there are going to be 5 or 6 people, and need it to work for both.
   - You want one rhyme that ends at the last person for 5, 6, *or* 7 people in the group. (This will be a long rhyme, but you're just looking for the number.)