Strategic Game Theory
Games!
Board games!
Sport games!
Video games!
More video games!
Politics and Finance
Today: play a series of games, see if we can figure out how to win!
Game 1: 21

- Two players take turns counting
- Start from “1”
- Each player can count at most three numbers
- First player that says “21” wins
Game 1: 21

- Two players take turns counting
- Start from “1”
- Each player can count at most three numbers
- First player that says “21” wins
  - Alice: 1, 2, 3
  - Bob: 4, 5
  - Alice: 6, 7
  - Bob: 8, 9, 10
  - Alice: 11
  - Bob: 12, 13, 14
  - Alice: 15, 16
  - Bob: 17, 18, 19
  - Alice: 20, 21
  - Alice wins
Let’s try it!
What’s the trick here?
Modification 1

- Same rules, but whoever first counts to “22” wins
Modification 1

- Same rules, but whoever first counts to “22” wins
- What about “23”?
Modification 1

- Same rules, but whoever first counts to "22" wins
- What about "23"?
- What about "24"? Who should go first?
Modification 1

- Same rules, but whoever first counts to “22” wins
- What about “23”?  
- What about “24”? Who should go first?  
- What about “2023”?  

Modification 2

- Same rules, but now each player can count up to four numbers each time
Modification 2

- Same rules, but now each player can count up to four numbers each time
- What about five numbers each time?
Final test for Game 1

- Each player can count up to nine numbers, first one to “2023” wins
Game 2: Grid game

- 4 × 5 board
- Start by placing a rock at the bottom left
- Each turn, a player can move the rock directly above, directly to the right, or diagonally up and above
- Whoever moves the rock at the top right corner wins
Game 2 modification

- What about a $5 \times 5$ board?
Game 2 modification

- What about a $5 \times 5$ board?
- What about a $8 \times 8$ board?
Game 2 modification

- What about a $5 \times 5$ board?
- What about a $8 \times 8$ board?
- What about a $100 \times 100$ board?
Game 3: Piles of stones

- Two piles of stones: 5, 7
- Each turn, a player can pick a pile, and take any number of stones from it
- Player who takes the last stone wins
Game 3 modification

- Three piles of stones: 3, 5, 7
- Player who takes the last stone wins