Numbers and Shapes
What numbers are...

e 1 2 0

8 3 π

42 -2

\frac{1}{2} 0.123

+1 \div \ldots
what numbers do...
what numbers do ...
What numbers do...

-1 as an action (i.e. multiply)

-1  0  1

↑ position
What motion, if done twice, does what \((-1)\) does?
Complex Numbers

\[ i \cdot i = -1 \]

\[ 2i \cdot 3 = 6i \]

\[ (2+i) \cdot i = 2i + i^2 \]

\[ = 2i - 1 \]
Complex numbers...

\[(1+3i) \cdot (5+7i) = 1 \cdot (5+7i) + 3i \cdot (5+7i)\]
\[= 5+7i + 15i + 21i^2\]
\[= (5-21) + 22i\]
\[= -16 + 22i\]

\[(1+i)(1-i) = 1 - i^2 + i(1-i)\]
\[= 1 + 1 = 2\]

\[\frac{1}{1+i} \cdot \frac{(1-i)}{(1-i)} = \frac{1-i}{2}\]
Complex #s Review:

- multiplying rotates and scales

- a complex number of length I can be thought of as:
  - a rotation
  - a position on the circle.
... Stand up!

1 piece of info!
# of pieces of info to describe a pt. on the sphere = 2

# ___ ... a rotm? = 3
New numbers are ...

\[ i^2 = -1 \]

\[ j^2 = -1 \]

\[ ij = k \]

\[ \pm 1 \cdot \text{length to scale by} \]
\( (3+2i+j+6k) \cdot (2+\frac{1}{2}i + \ldots) = \_ \_ \_ \)
Here as he walked by
on the 16th of October 1843
Sir William Rowan Hamilton
in a flash of genius discovered
the fundamental formula for
quaternion multiplication
\[ i^2 = j^2 = k^2 = ijk = -1 \]
& cut it on a stone of this bridge
New #’s: quaternions

\[ a + bi + c j + d k \]
New numbers do?

- position in 3-d space, recorded as

\[ xi + yj + zk \]

- actions correspond to quaternions

\[ q = a + bi + cj + dk \]
\( x_i + y_j + z_k \)

\[ \varphi \cdot (x_i + y_j + z_k) \frac{1}{\varphi} \]
Real #s \ldots \quad \mathbb{R}

Complex #s \ldots \quad \mathbb{C}

Quaternions \ldots \quad \mathbb{H}

Octonions \ldots \quad \mathbb{O}

\begin{align*}
\text{1} & \quad \text{1} \\
\text{2} & \quad \text{2} \\
\text{4} & \quad \text{8}
\end{align*}
<table>
<thead>
<tr>
<th>R</th>
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... what's next?
There is no arithmetic of $n$-tuples except when $n = 1, 2, 4, 8$. (Bott, Kervaire, Milnor)