Finding patterns in differences

The mathematics behind the game SET
What is SET?
What is SET?

Is there a card that doesn’t belong?
What is SET?

Is there a card that doesn’t belong?

Yes
What is SET?

Is there a card that doesn’t belong?

Yes

Not a SET!
What is SET?

Is there a card that doesn’t belong?

- Yes: Not a SET!
- No: A SET!
What is SET?

Is there a card that doesn’t belong?
What is SET?

Is there a card that doesn’t belong?
What is SET?

Is there a card that doesn’t belong?

NOT A SET!
What is SET?

Is there a card that doesn’t belong?
What is SET?

Is there a card that doesn’t belong?
What is SET?

Is there a card that doesn’t belong?

NOT A SET!
What is SET?

Is there a card that doesn’t belong?
What is SET?

Is there a card that doesn’t belong?

Yes, the second card does not belong.
What is SET?

Is there a card that doesn’t belong?

A SET!!
What is SET?

Is there a card that doesn’t belong?
What is SET?

Is there a card that doesn’t belong?
What is SET?

Is there a card that doesn’t belong?

A SET!!
What is SET?

Is there a card that doesn’t belong?
What is SET?

Is there a card that doesn’t belong?
What is SET?

Is there a card that doesn’t belong?

A SET!!
Each card in SET has 4 properties

<table>
<thead>
<tr>
<th>Color</th>
<th>Number</th>
<th>Shading</th>
<th>Shape</th>
</tr>
</thead>
<tbody>
<tr>
<td>![Color]</td>
<td>![Number]</td>
<td>![Shading]</td>
<td>![Shape]</td>
</tr>
<tr>
<td>![Color]</td>
<td>![Number]</td>
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</tbody>
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<tr>
<th>Color</th>
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<th>Shape</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Color 1" /></td>
<td><img src="image2" alt="Number 1" /></td>
<td><img src="image3" alt="Shading 1" /></td>
<td><img src="image4" alt="Shape 1" /></td>
</tr>
<tr>
<td><img src="image5" alt="Color 2" /></td>
<td><img src="image6" alt="Number 2" /></td>
<td><img src="image7" alt="Shading 2" /></td>
<td><img src="image8" alt="Shape 2" /></td>
</tr>
<tr>
<td><img src="image9" alt="Color 3" /></td>
<td><img src="image10" alt="Number 3" /></td>
<td><img src="image11" alt="Shading 3" /></td>
<td><img src="image12" alt="Shape 3" /></td>
</tr>
</tbody>
</table>

Have $3 \times 3 \times 3 \times 3 = 3^4 = 81$ cards
No repeats!
"No card that doesn't belong"

\[ \Leftrightarrow \]

"For every property, all are the same or all are different"
For every property, all are the same or all are different
For every property, all are the same or all are different

Two are red, one is green
For every property, all are the same or all are different

Two are red, one is green

NOT A SET!
For every property, all are the same or all are different.
For every property, all are the same or all are different

Two are solid, one is empty
For every property, all are the same or all are different

Two are solid, one is empty

NOT A SET!
For every property, all are the same or all are different
For every property, all are the same or all are different

All solid, all green, all diamonds, all *different* numbers
For every property, all are the same or all are different

All solid, all green, all diamonds, all *different* numbers

A SET!!
For every property, all are the same or all are different
For every property, all are the same or all are different

All the same number, all solid, all different shapes, all different colors
For every property, all are the same or all are different

All the same number, all solid, all different shapes, all different colors

A SET!!
For every property, all are the same or all are different.
For every property, all are the same or all are different

Same number, all different shading, all different shapes, all different colors
For every property, all are the same or all are different

Same number, all different shading, all different shapes, all different colors

A SET!!
For every property, all are the same or all are different
For every property, all are the same or all are different

All different numbers, all different colors, all different shapes, all different shading
For every property, all are the same or all are different

All different numbers, all different colors, all different shapes, all different shading

A SET!!
How many SETs are there?
What are the possible SETs?
What are the possible SETs?

Are there more?
What are the possible SETs?
What are the possible SETs?
What are the possible SETs?
What are the possible SETs?
What are the possible SETs?
What are the possible SETs?
We observed that SETs lie on lines!
Is this a real thing?!!?
Is this a real thing?!?

Why can we just repeat the grid like that?
Is this a real thing?!?

Why can we just repeat the grid like that?

Does every SET have to lie on a line? Or is it just the ones that we found?
Why can we just repeat the grid like that?
Why can we just repeat the grid like that?
Why can we just repeat the grid like that?
Why can we just repeat the grid like that?
14 is the same as 2
26 is the same as 2
14 is the same as 2
26 is the same as 2
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26 is the same as 2

This is called working $\text{mod } 12$.
We do usual arithmetic, but treat 12 as 0.
14 is the same as 2
26 is the same as 2

This is called working \( \text{mod 12} \).
We do usual arithmetic, but treat 12 as 0.
14 is the same as 2
26 is the same as 2

This is called working $\text{mod } 12$.
We do usual arithmetic, but treat 12 as 0.

It makes sense to repeat when we work $\text{mod } 3$!
Is this a real thing?!?

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Weird but AMAZING thing about mod 3 arithmetic:

The only possible slopes are 1, 0, -1, or $\infty$.
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Weird but AMAZING thing about mod 3 arithmetic:

The only possible slopes are $1$, $0$, $-1$, or $\infty$.
Weird but AMAZING thing about mod 3 arithmetic:

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Weird but AMAZING thing about mod 3 arithmetic:

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Slopes are 1, 1, 1
Does every SET have to lie on a line? Or is it just the ones that we found?

**SET**
- slopes are 1, 1, 1

**NOT A SET**
- slopes are 0, 1, -1
Does every SET have to lie on a line? Or is it just the ones that we found?

If 3 points lie on a line, then all pairs have the same slope.
If 3 points do not lie on a line, then all pairs have different slopes.
Does every SET have to lie on a line? Or is it just the ones that we found?
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Weird but AMAZING thing about mod 3 arithmetic:

The only possible slopes are 1, 0, -1, or \( \infty \).

If you pick 3 different numbers from 1, 0, -1, or \( \infty \), you must end up with a 0 or \( \infty \)!
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