



Make sure to grab the handouts!
Go ahead and start with question 1.

Mathematical Tiling and Organization



Dr. Brandy Wiegiers
Central Washington University



+ Math Circle Promise

I solemnly swear

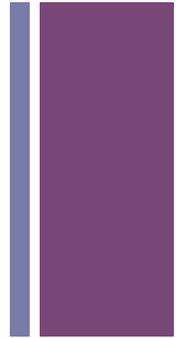
that if I already know the answer

to the math problem

I will not yell it out loud

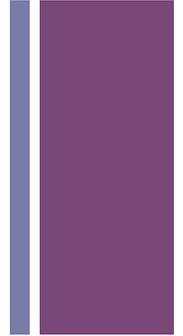
so those around me

can experience the joy of learning.



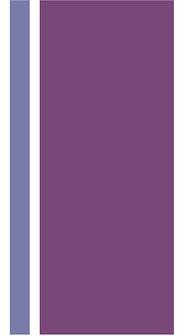


WHAT IS A MATHEMATICAL TILING?

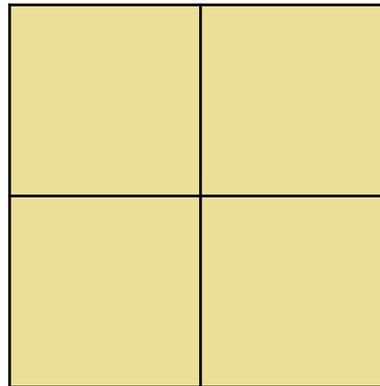
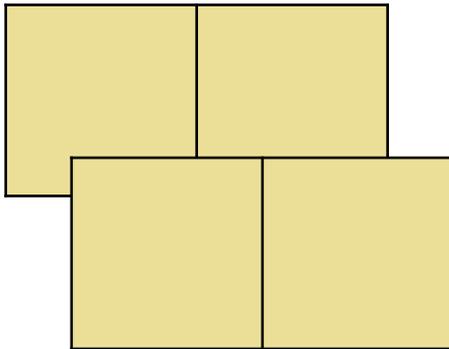




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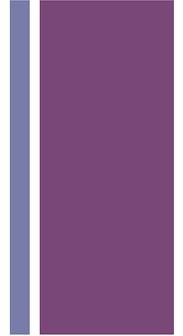
- Like your bathroom tiles...
 - All the tiles sit next to each other, sides touching



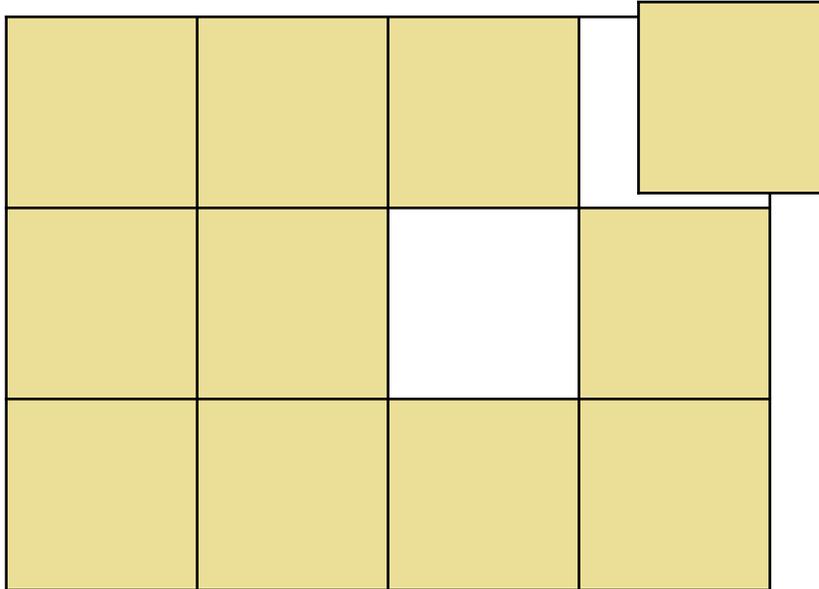
- Can't skip any spaces- need to cover every spot



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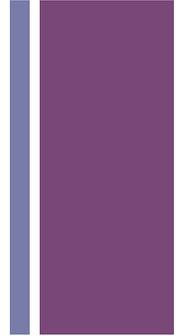
- Like your bathroom tiles...
 - All the tiles sit next to each other, sides touching
 - Can't skip any spaces- need to cover every spot on the "board"



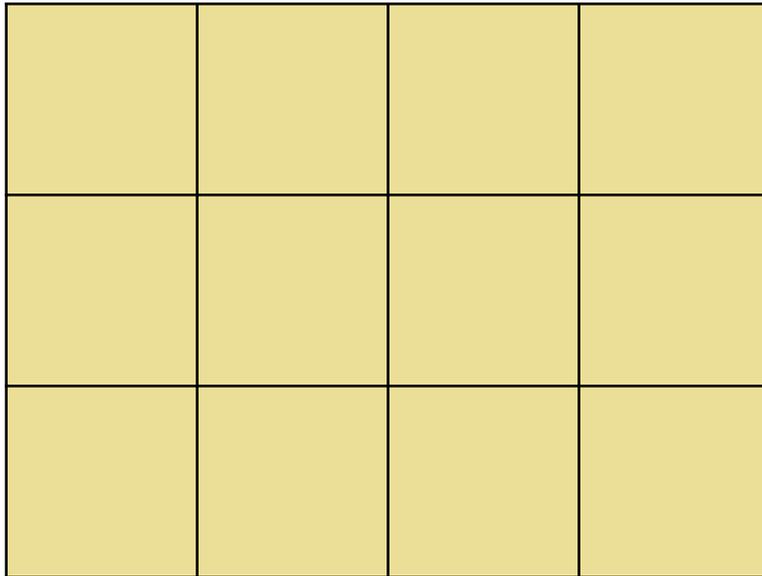
- But sometimes we can add holes in our "board"



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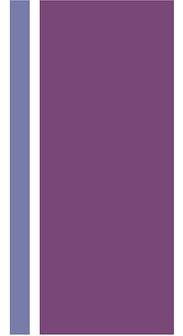
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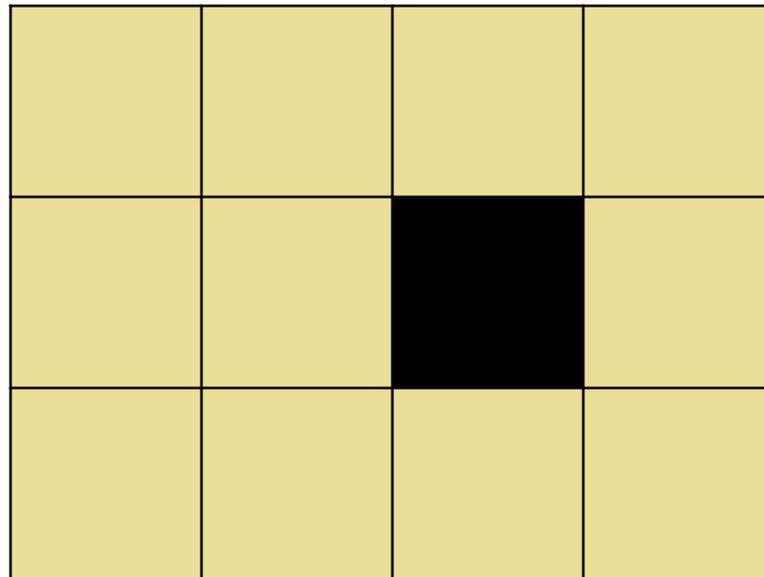
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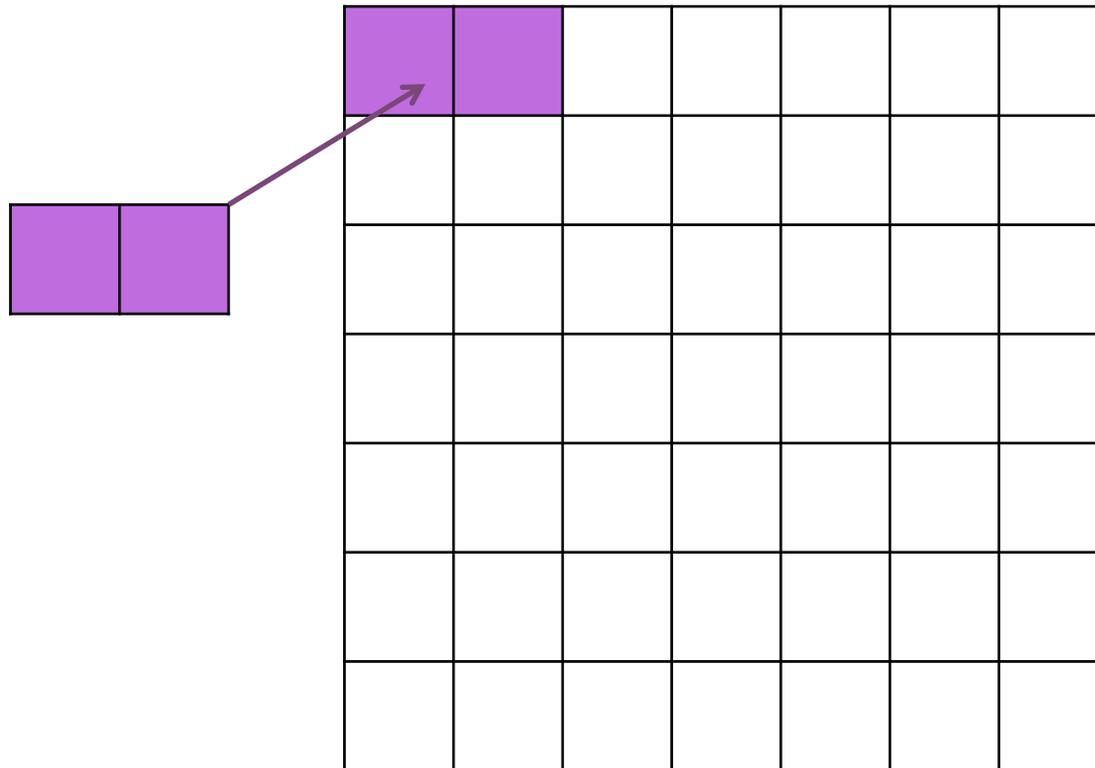
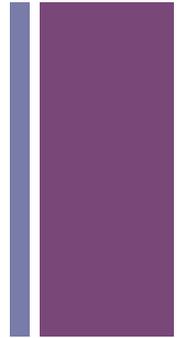
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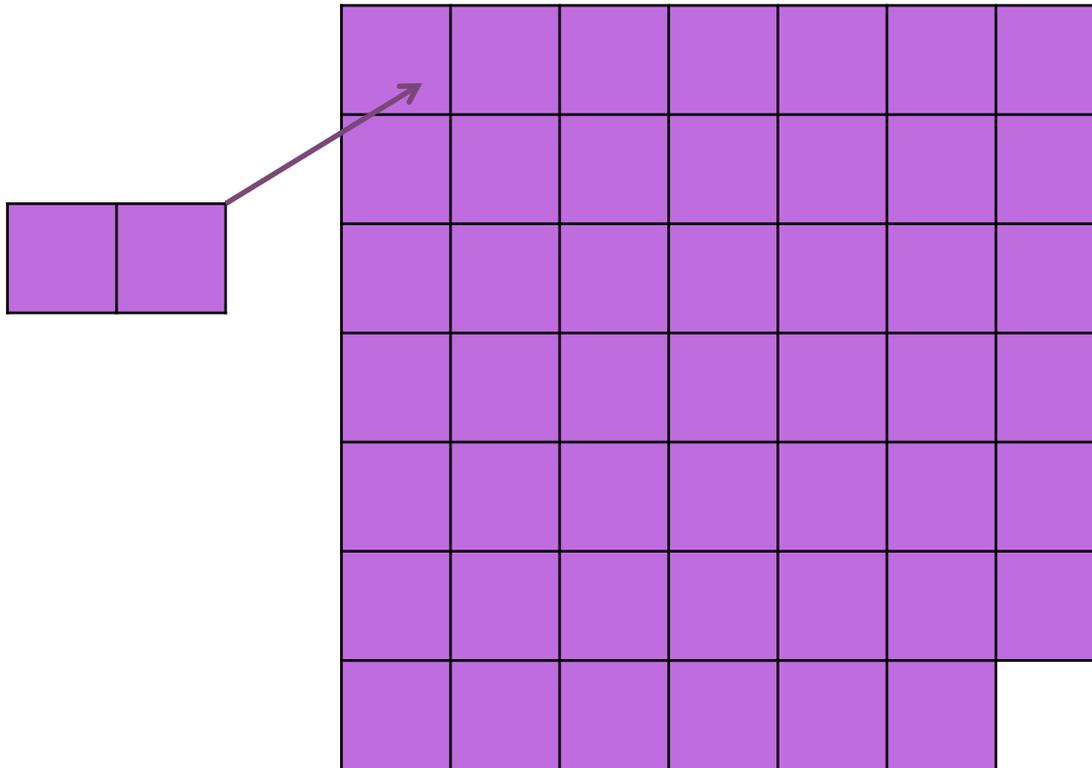
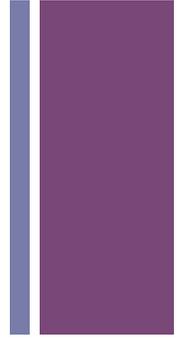
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 - All the tiles sit next to each other, sides touching
 - Can't skip any spaces- need to cover every spot on the "board"
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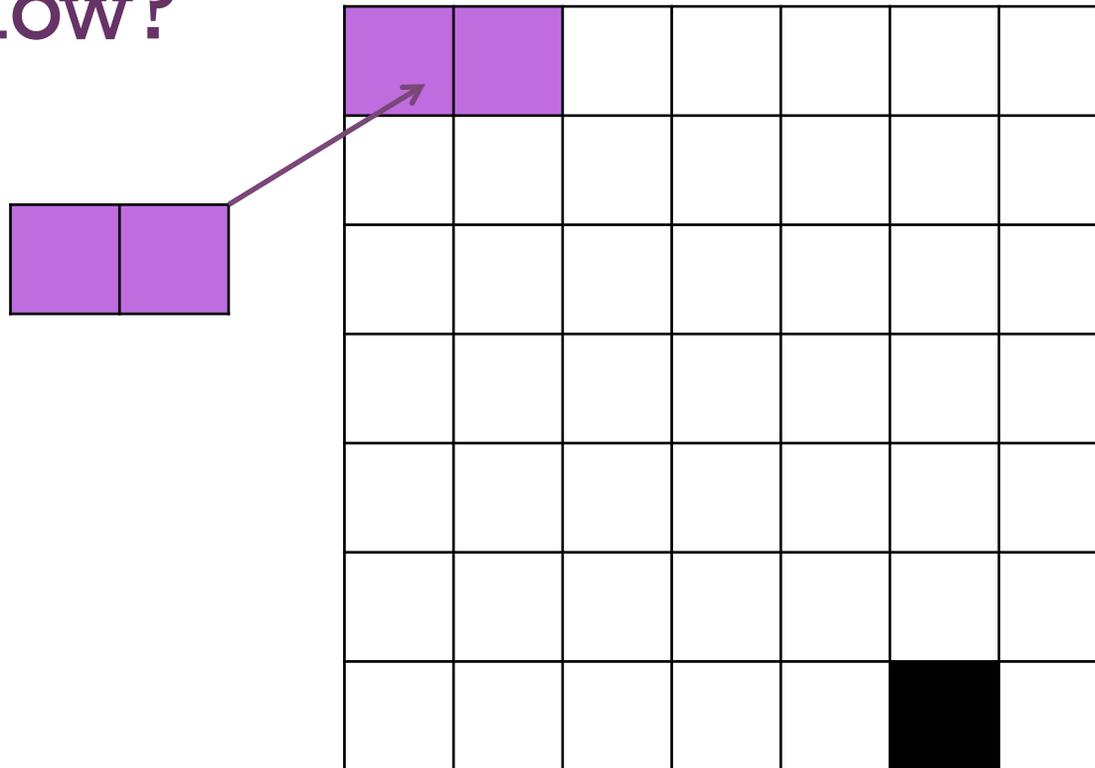
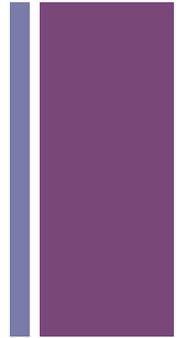
+ Question 1: Is it possible to tile a 7×7 board with 2×1 tiles?



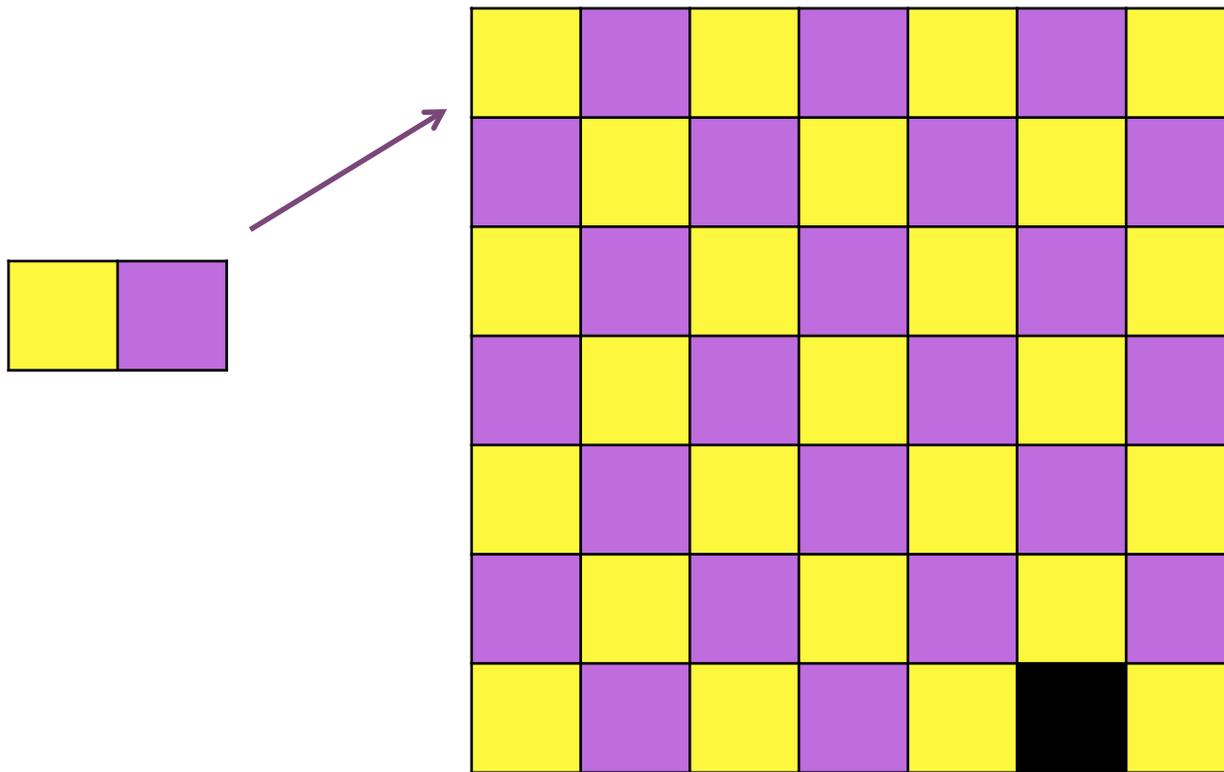
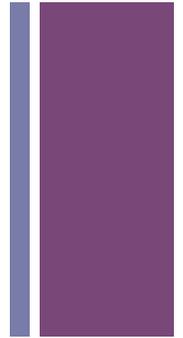
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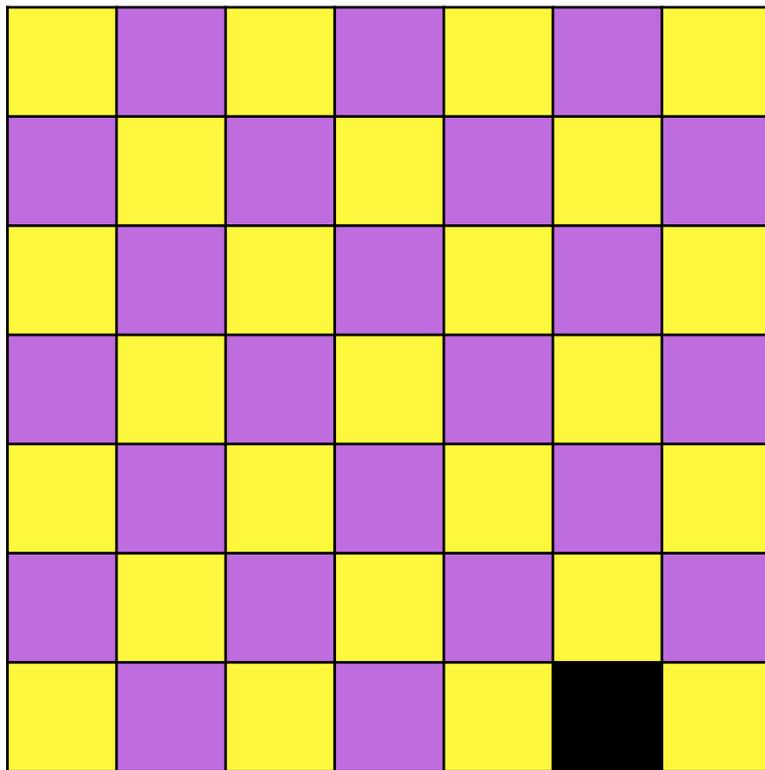
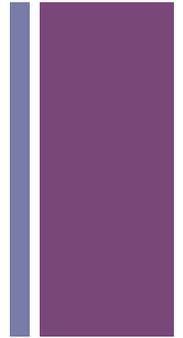
+ Question 1.5: Is it possible to tile a 7x7 board with 2x1 tiles given the 1 hole in the board at the spot below?



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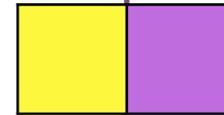


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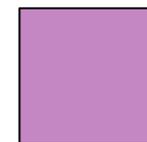
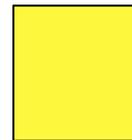
How many yellow boxes are there in a domino?

How many purple boxes are there in a domino?

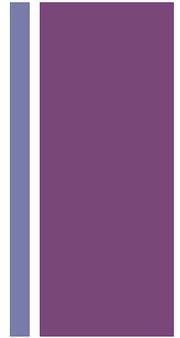


How many yellow boxes are there in the grid?

How many purple boxes are there in the grid?



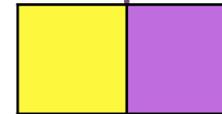
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1	1	2	2	3	3	4
4	5	5	6	6	7	7
8	8	9	9	10	10	11
11	12	12	13	13	14	14
15	15	16	16	17	17	18
18	19	19	20	20	21	21
22	22	23	23	24		25

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How many yellow boxes are there in the grid?

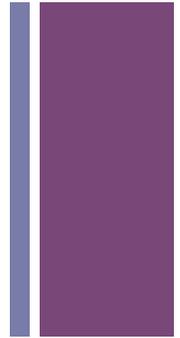
How many purple boxes are there in the grid?

25

23

How many dominoes can we place on the 7x7 grid with this hole?

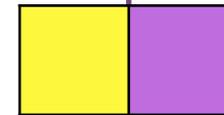
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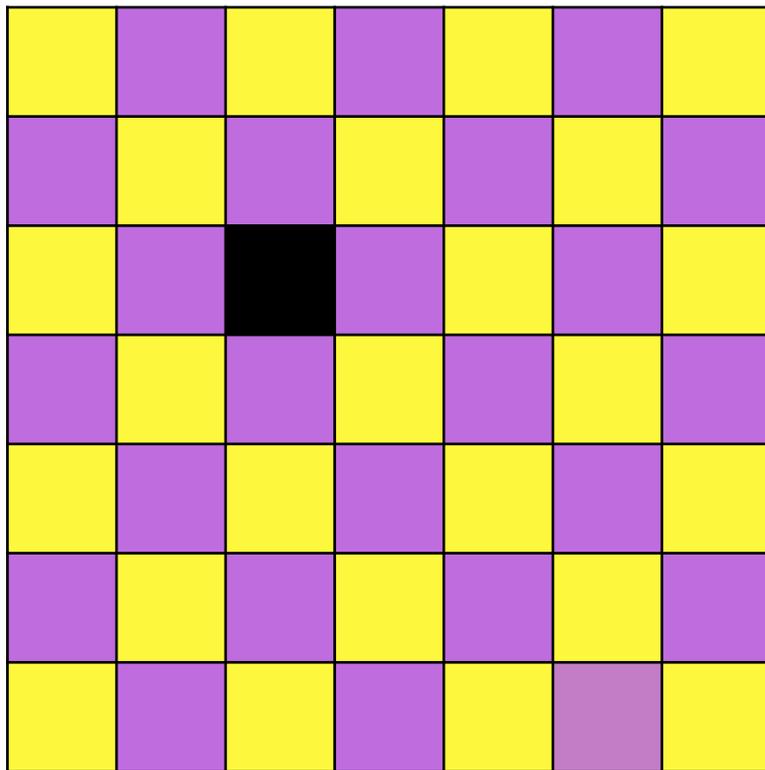
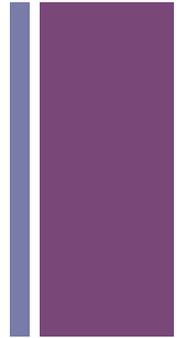
25

23

How many dominoes can we place on the 7x7 grid with this hole?

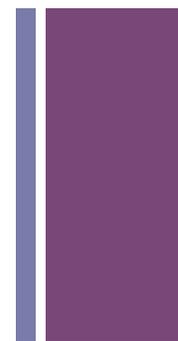
23

+ Question 1.75: Is it possible to tile a 7×7 board with 2×1 tiles given the 1 hole in the board at the spot below?



Can we cover the entire 7×7 board with this hole?

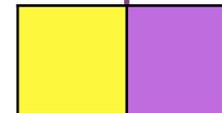
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How many yellow boxes are there in a domino?

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How many yellow boxes are there in the grid?

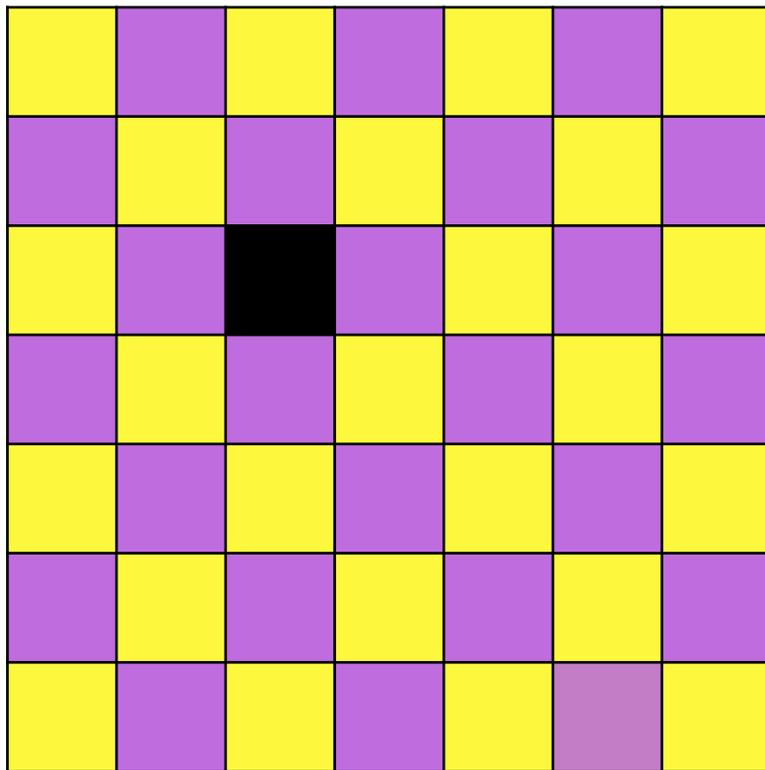
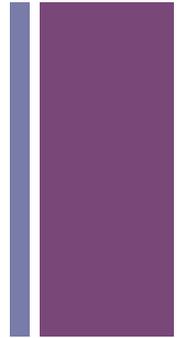
How many purple boxes are there in the grid?

24

24

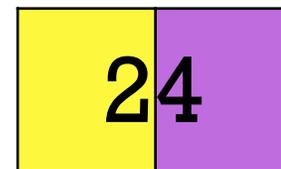
How many dominoes can we place on the 7x7 grid with this hole?

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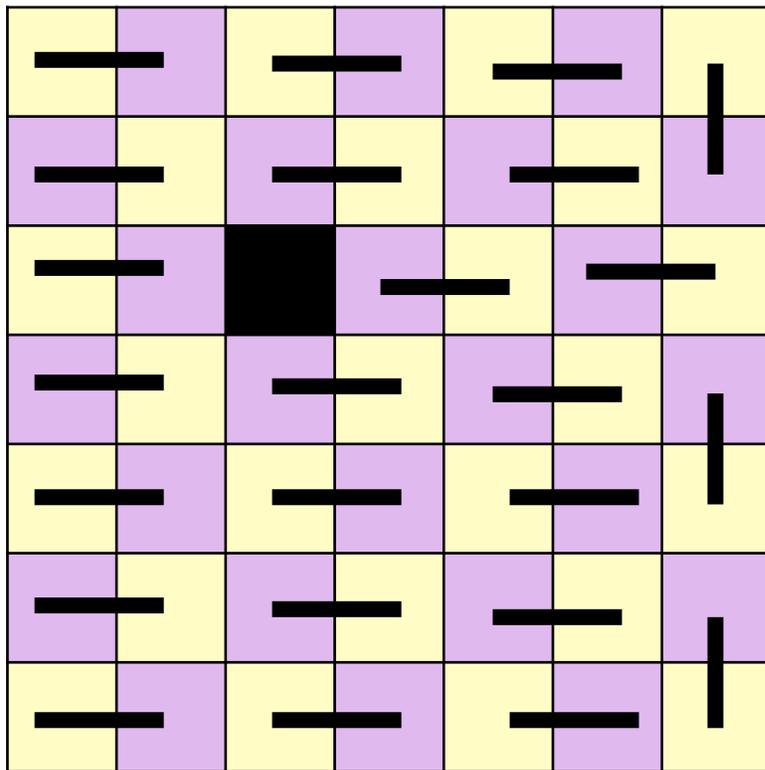
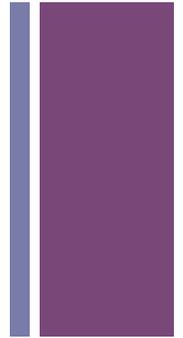


Can we cover the entire 7x7 board with this hole?

YES!

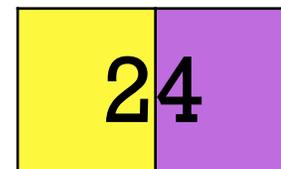


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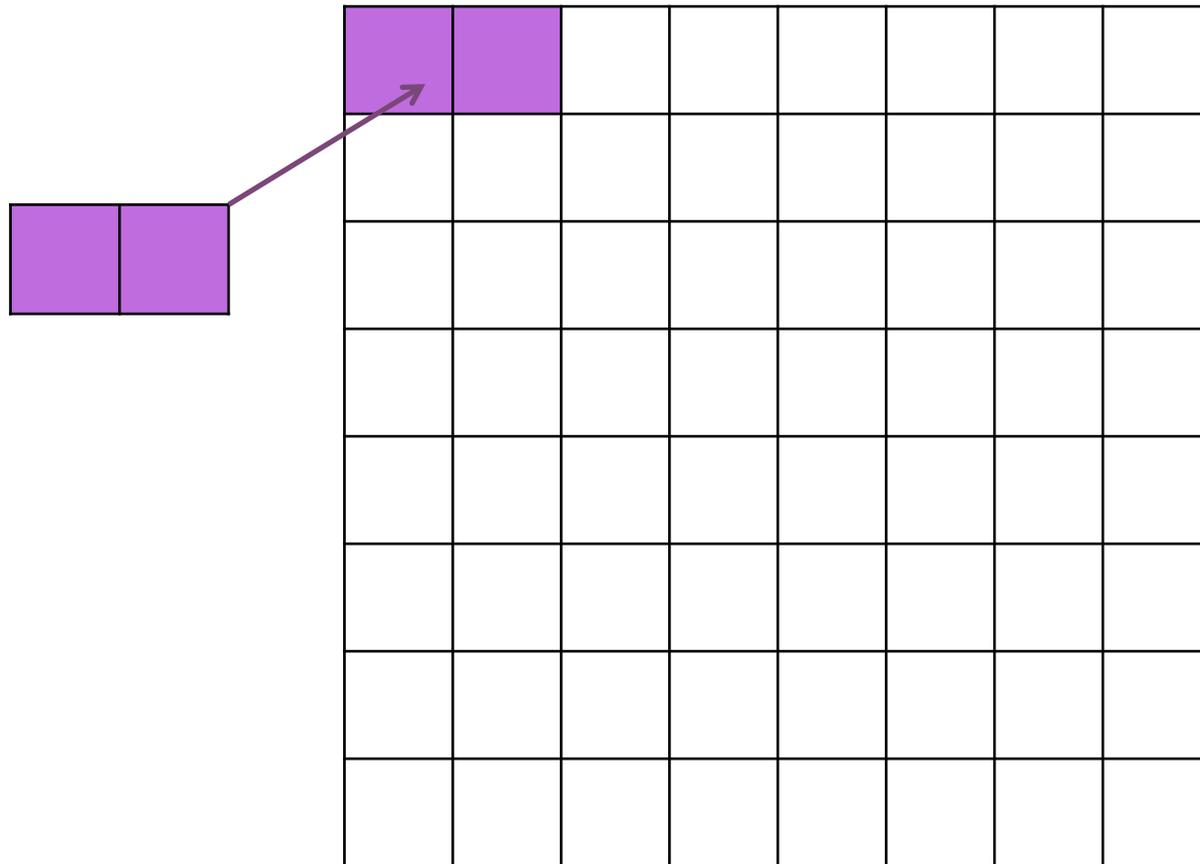
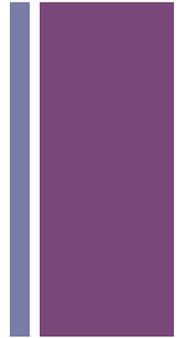


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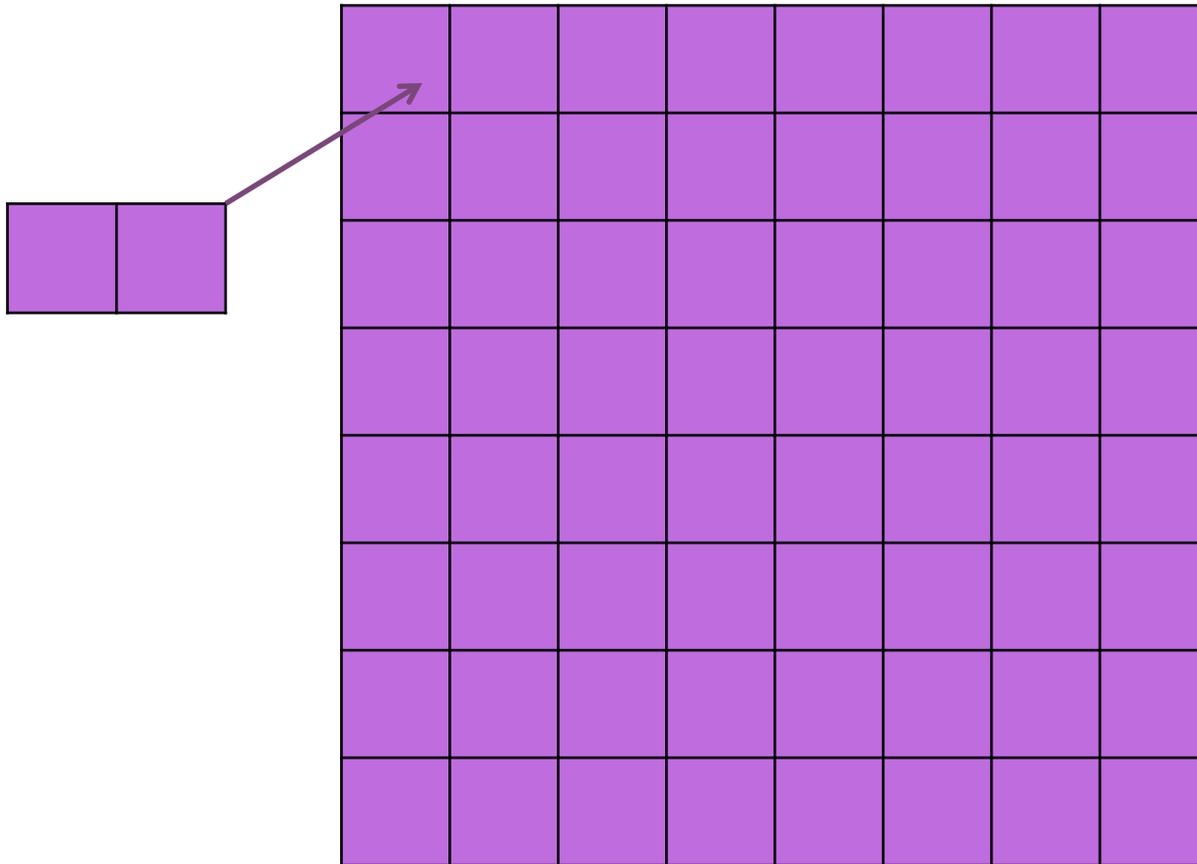
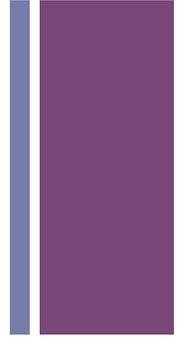
YES!



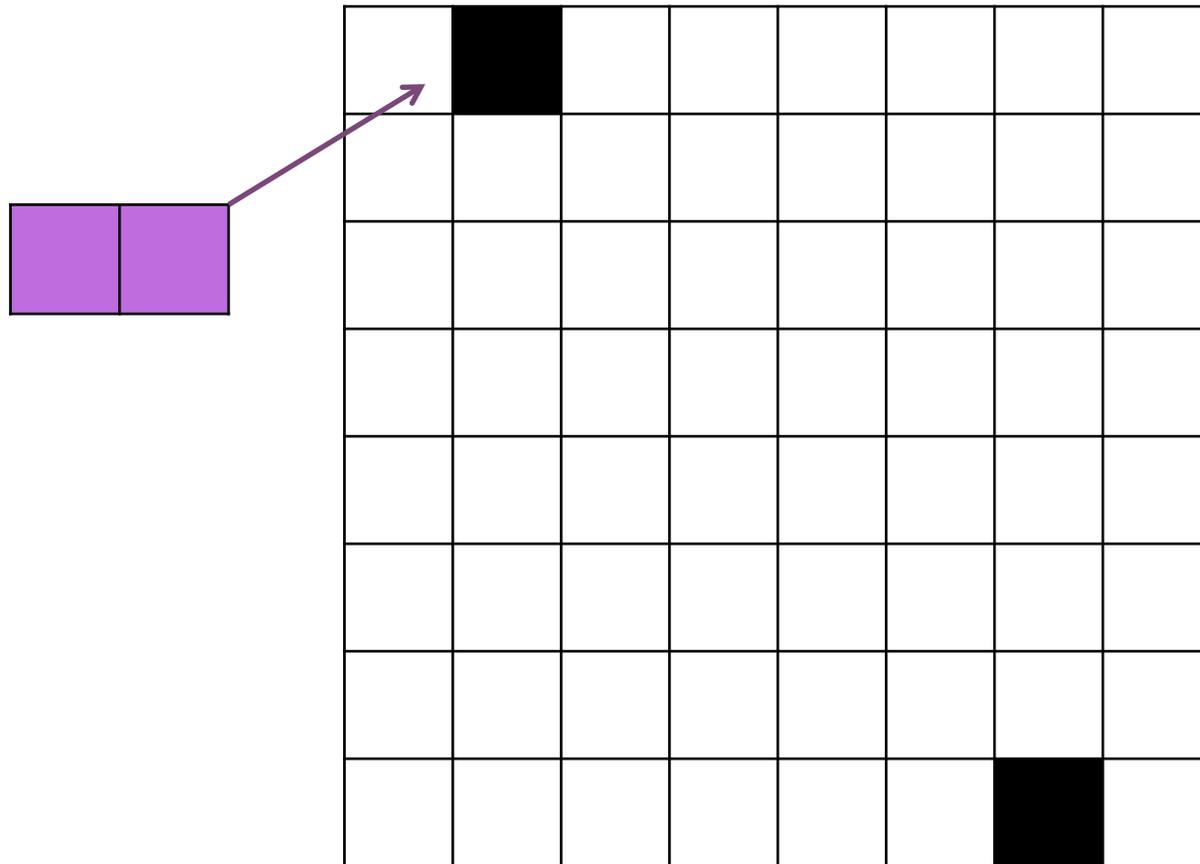
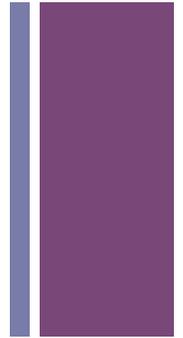
+ Question 2: Is it possible to tile a 8x8 board with 2x1 tiles?



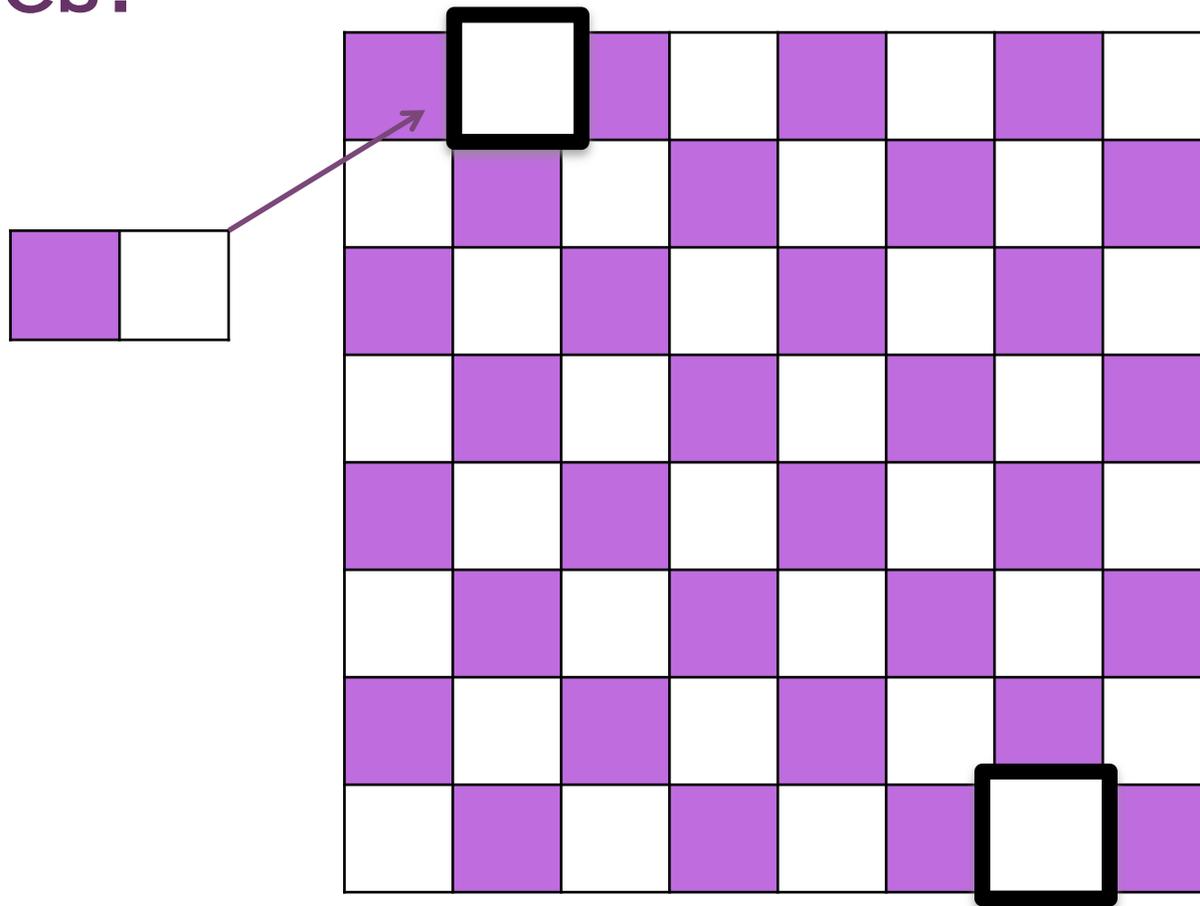
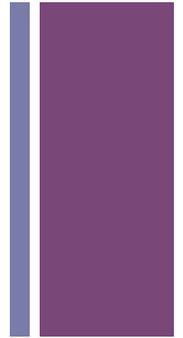
+ Question 2: Is it possible to tile a 8x8 board with 2x1 tiles?



+ Question 2.5: Is it possible to tile a
8x8 board with 2x1 tiles and 2
holes?

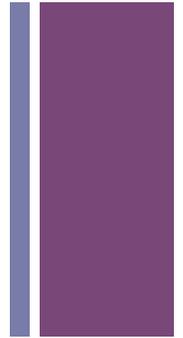


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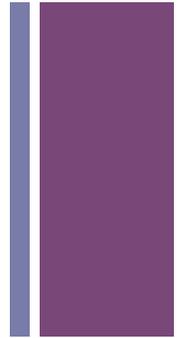
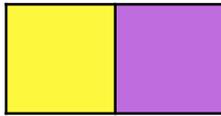


+ What's the next question?

- 7x7 board
- 8x8 board
- ??



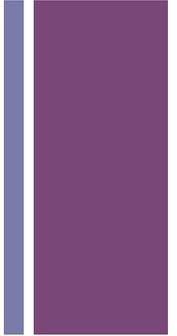
+ What's the next question?



- Q: In general, is it possible to tile an $n \times n$ board with 2×1 tiles? If so, which boards can you tile and why?
- Q: In general, if n is odd, is it possible to tile an $n \times n$ board with 2×1 tiles if one square is covered with a 1×1 tile? Does it matter which square is covered?
- Q: In general, if n is even, is it possible to tile an $n \times n$ board with 2×1 tiles if two squares are removed? Does it matter which two squares are removed?

+ What's the next question?

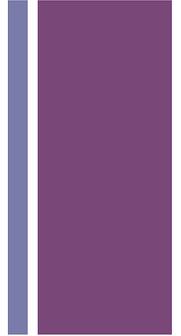
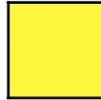
What if instead of focusing on domino problems we change our tile shapes?



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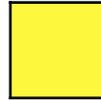
- Monominoes



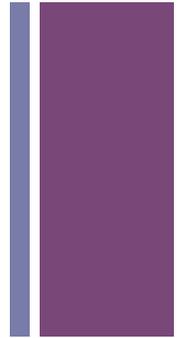
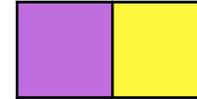
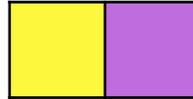
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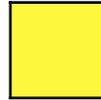
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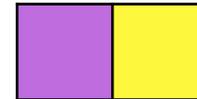
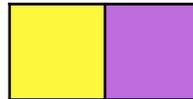
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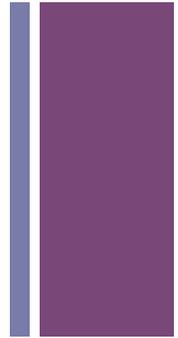
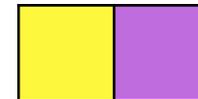


- Dominoes:



- But all of these are the same tile, rotated around.

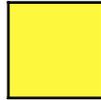
There is just 1 tile with 2 squares



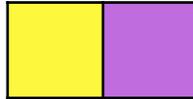
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- Monominoes



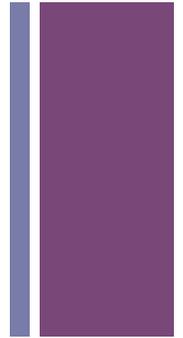
- Dominoes:



- Trominoes:

Q: How many Trominoes are there?

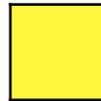
*Not counting rotations/ reflections of the same tile



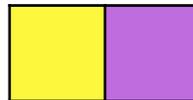
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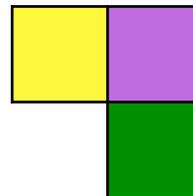
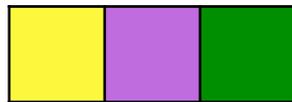
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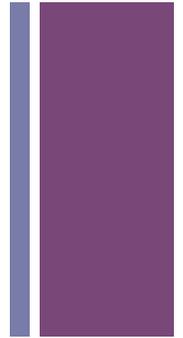
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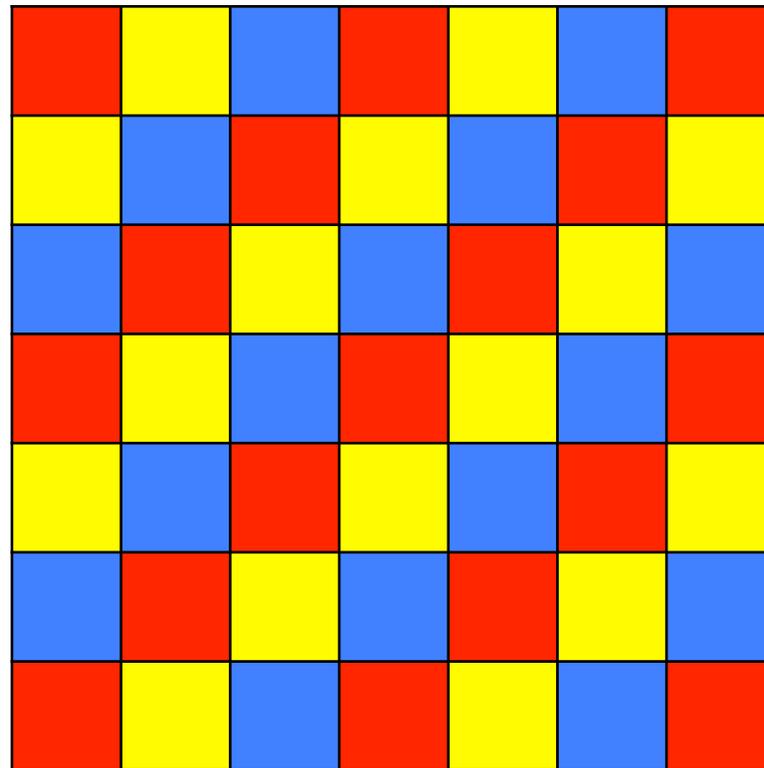
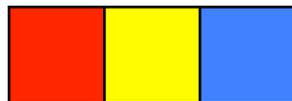
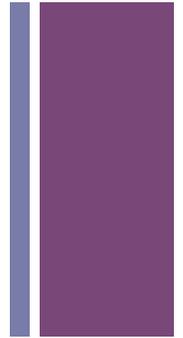
- Trominoes:



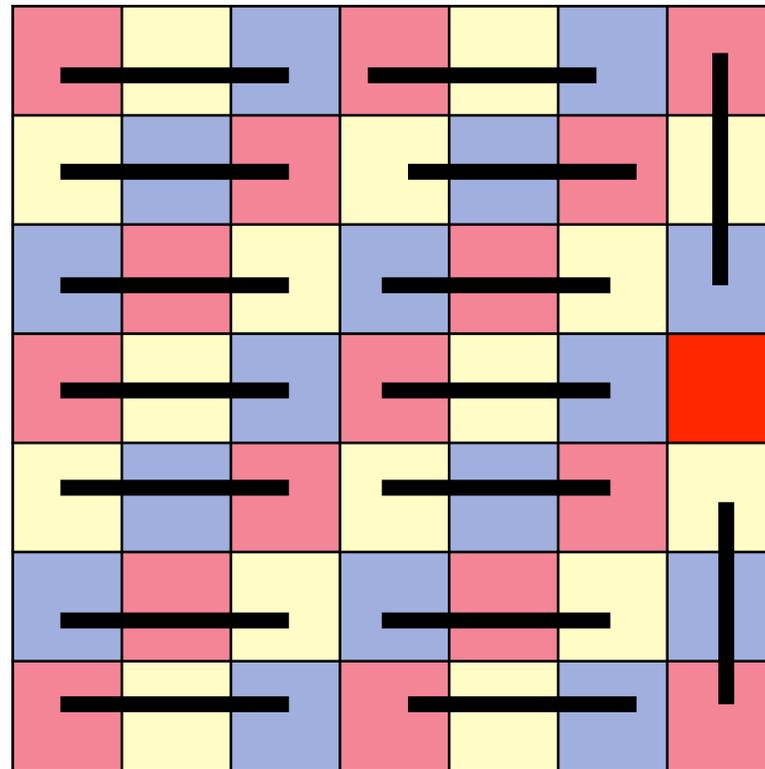
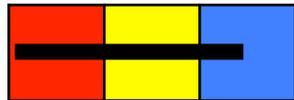
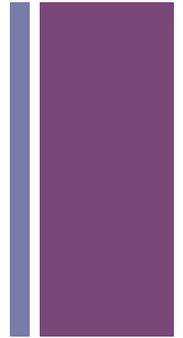
* colors indicate where we added the extra tiles.



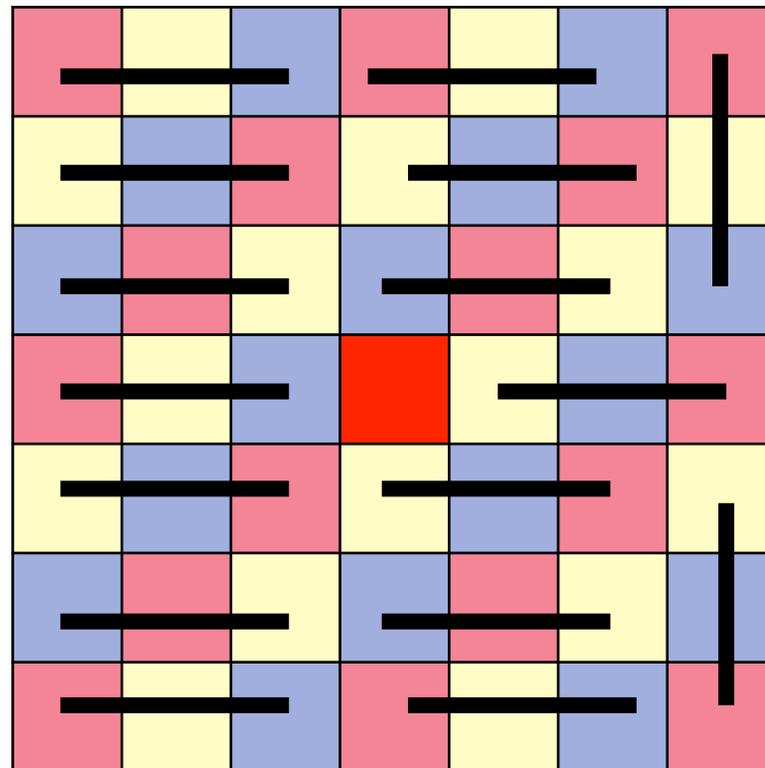
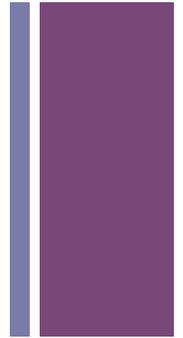
+ Q3: Is it possible to tile an 7x7 board with 16 3x1 tiles and 1 hole?



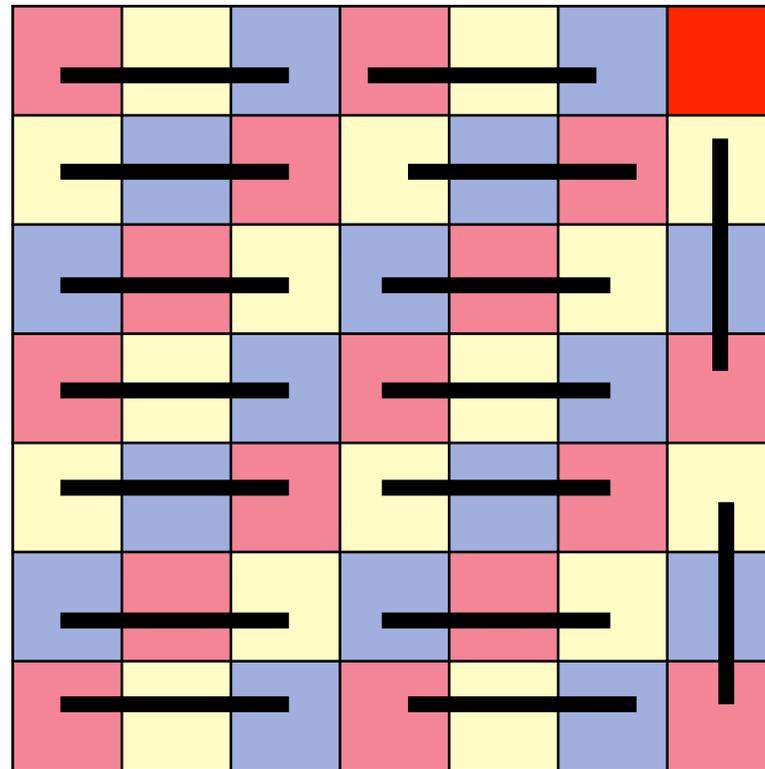
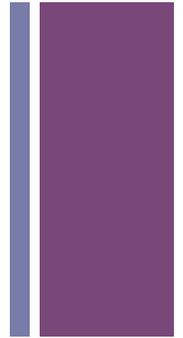
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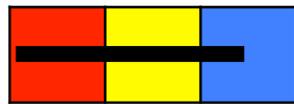
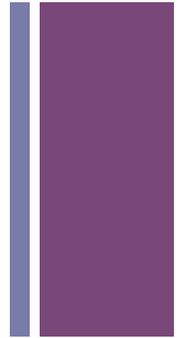
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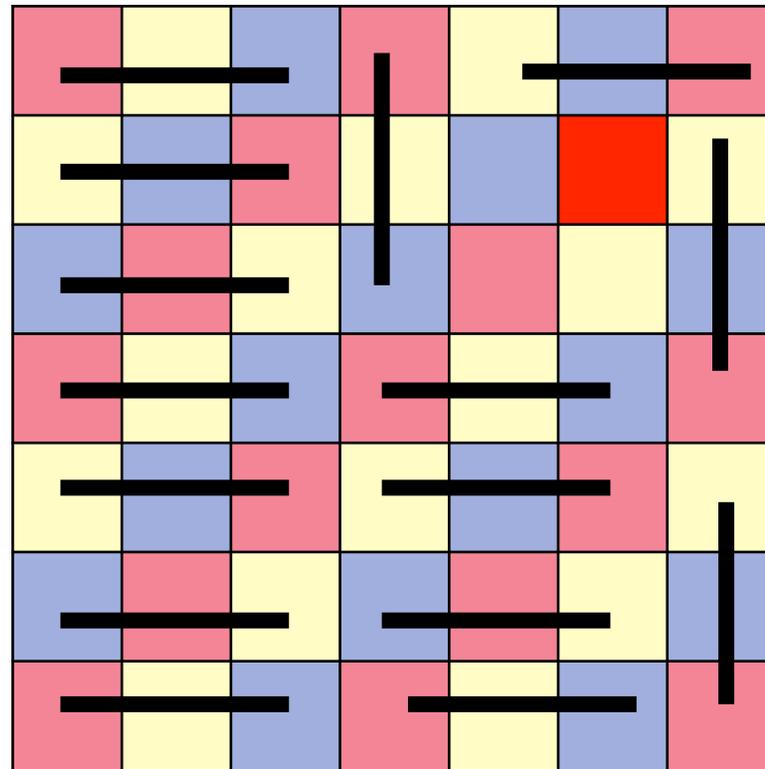
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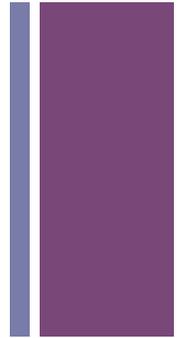
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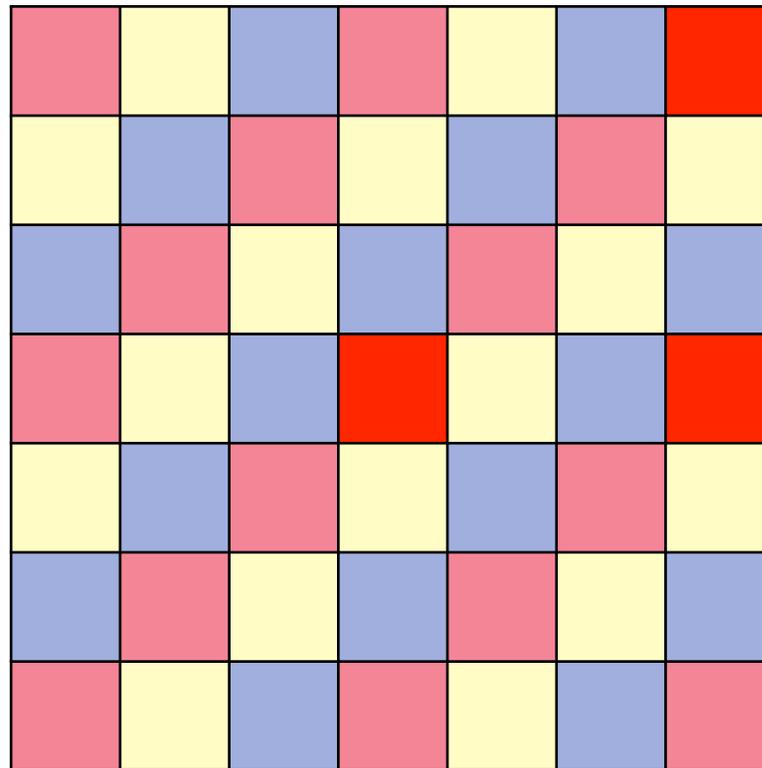
?



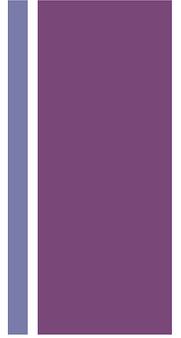
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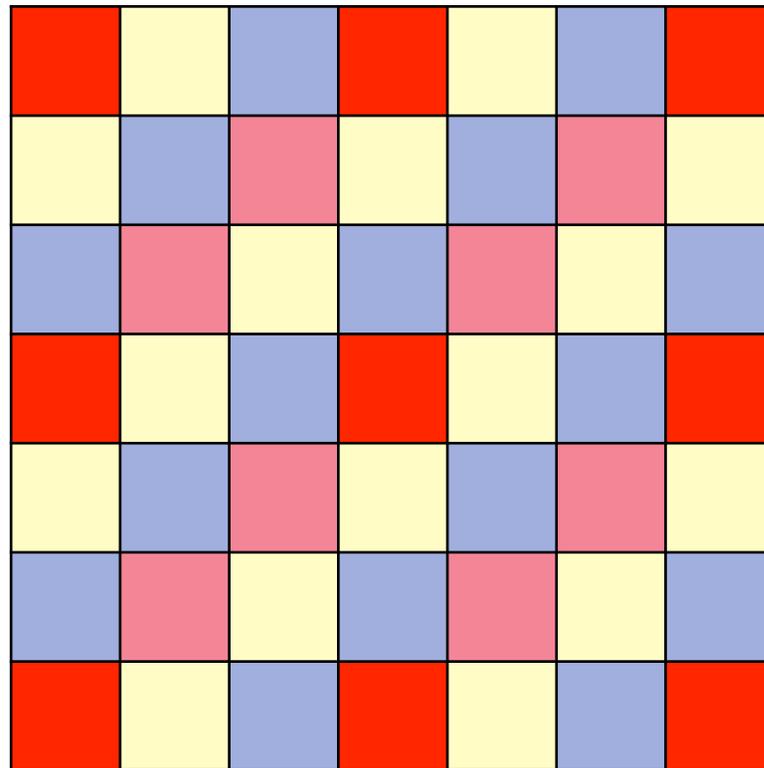
■ I know that these 3 work



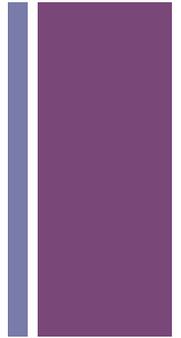
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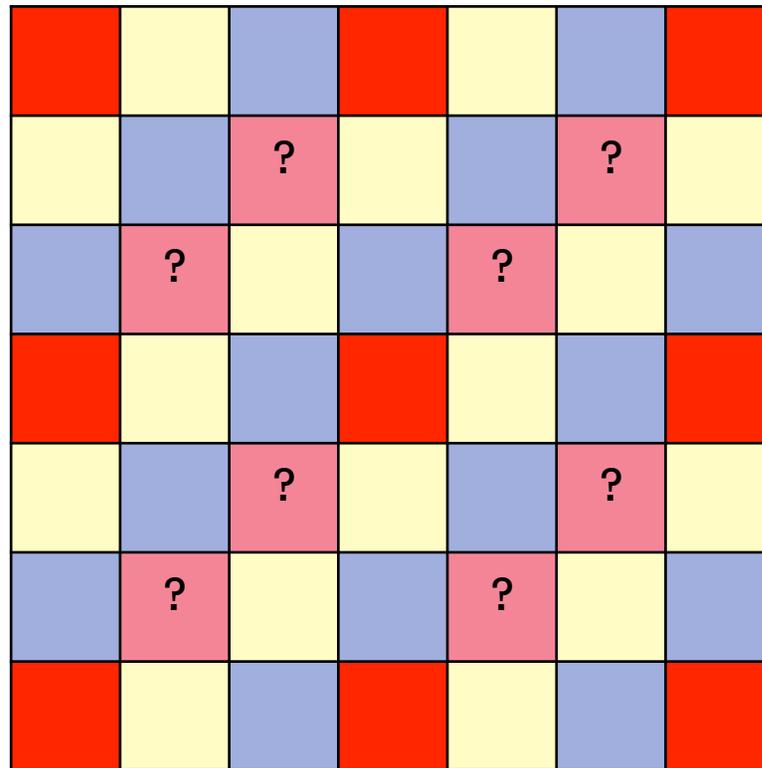
- I can then rotate the board and make a rotation argument for these same spots in the other corners/sides



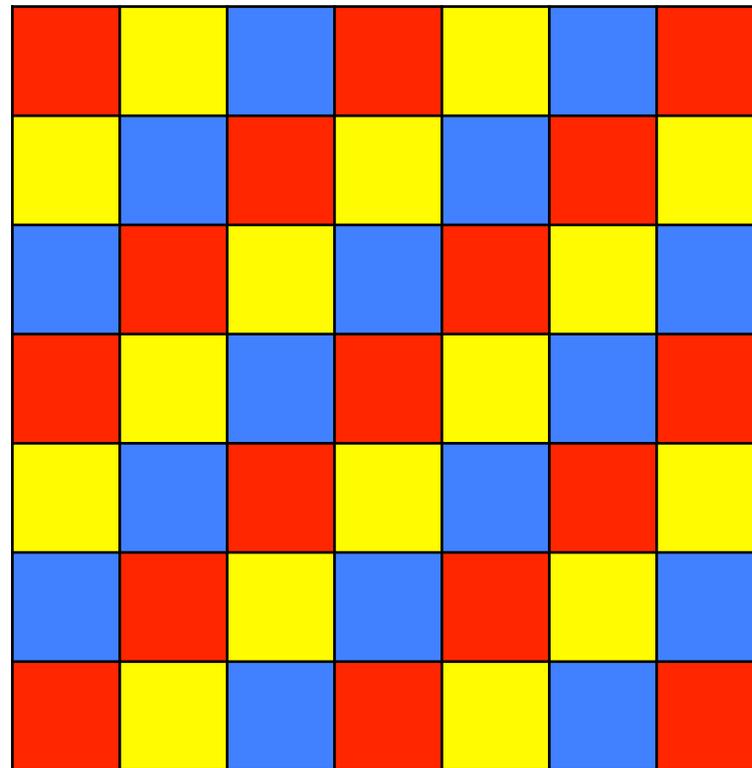
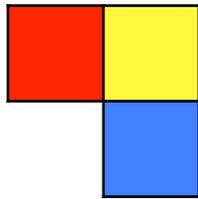
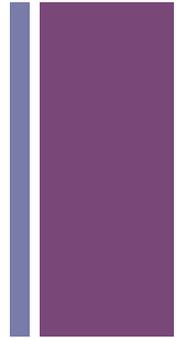
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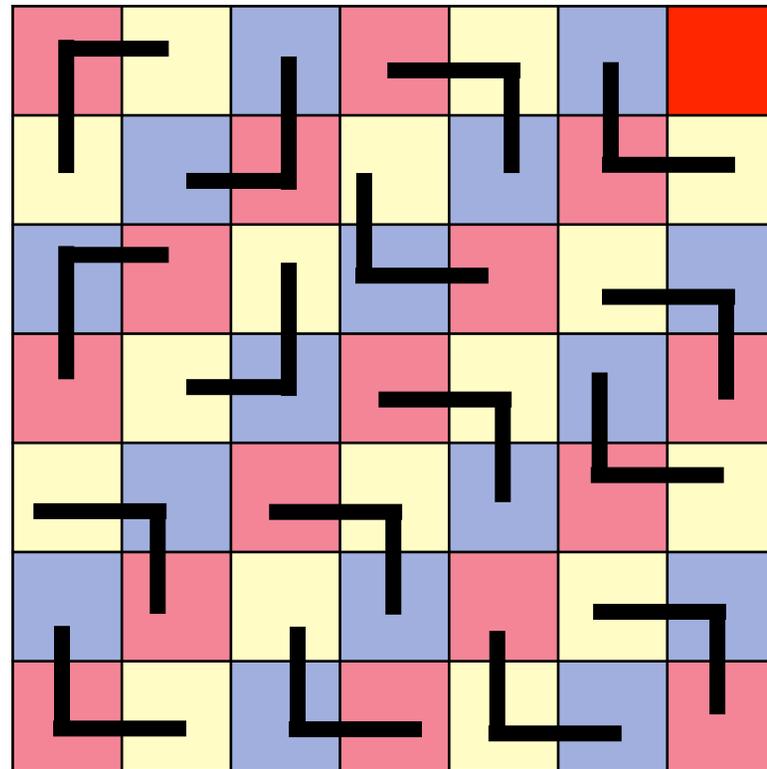
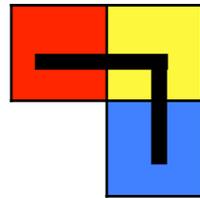
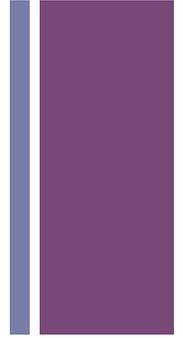
- I'm going to leave it to you to discover if the other inside points work



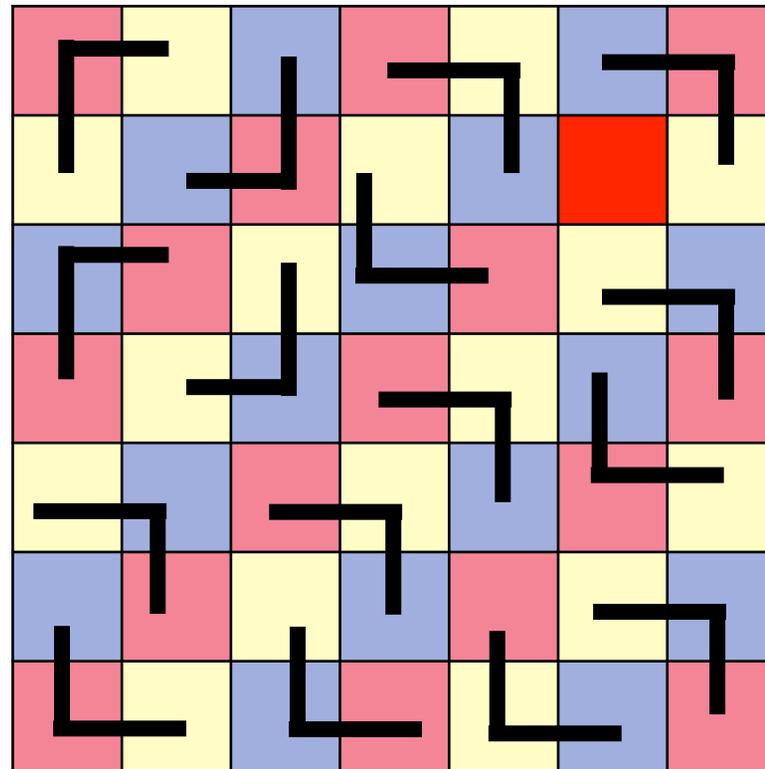
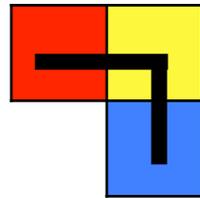
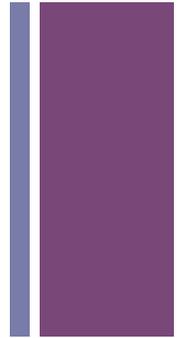
+ Q3.5 : Is it possible to tile an 7x7 board with 21 L-shaped trominoes and 1 hole?



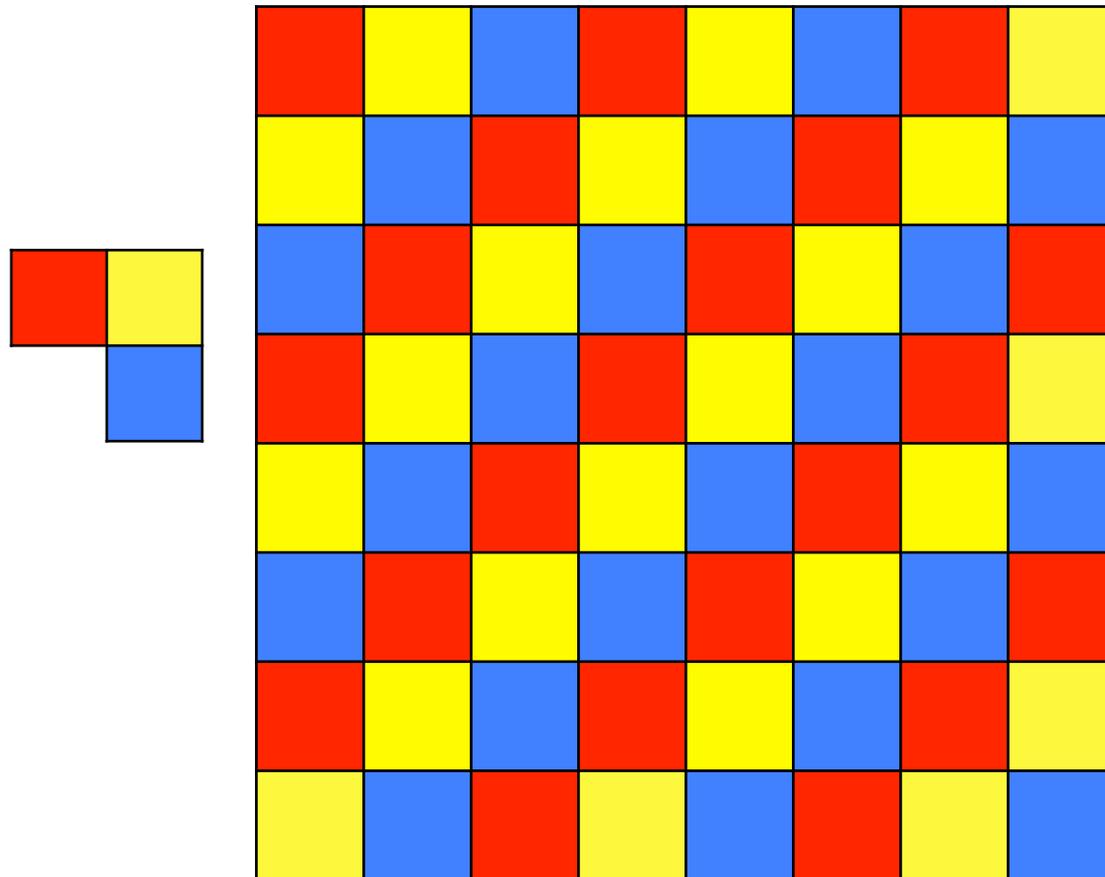
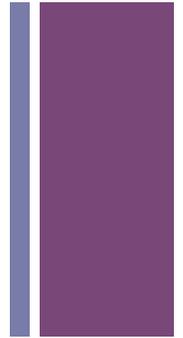
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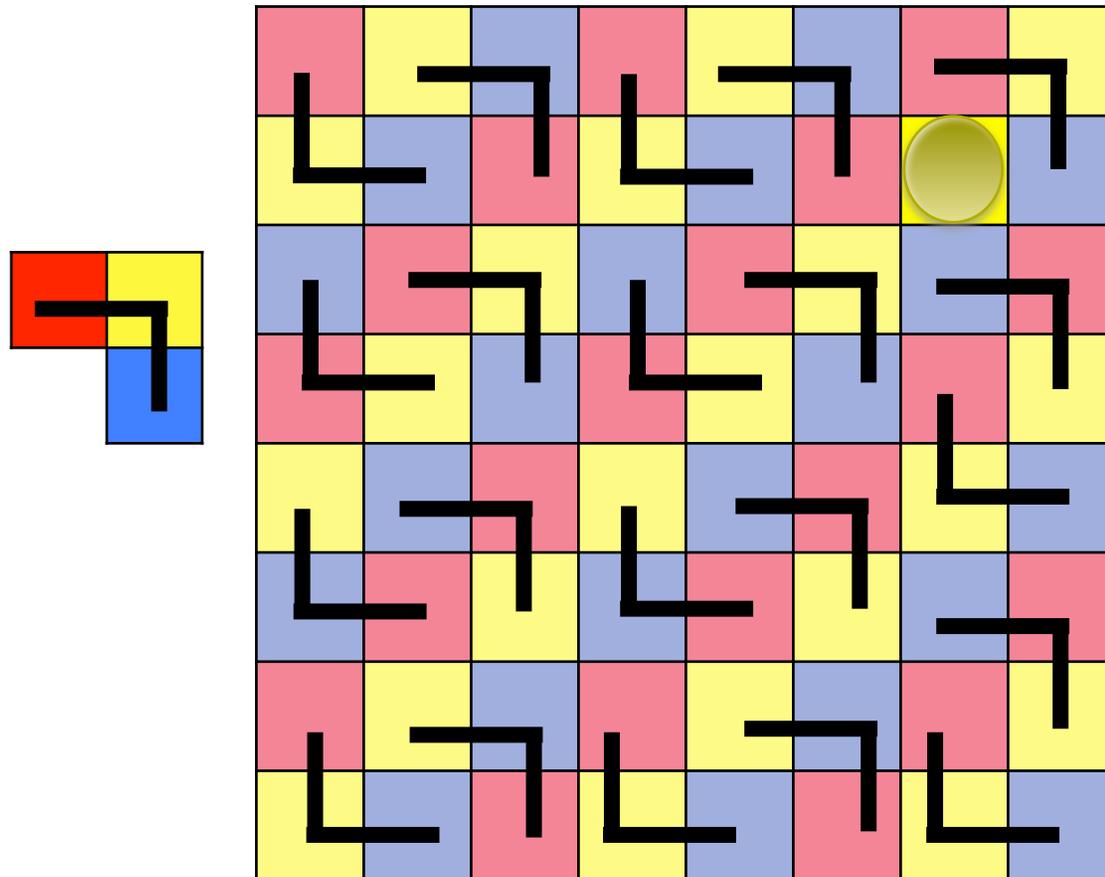
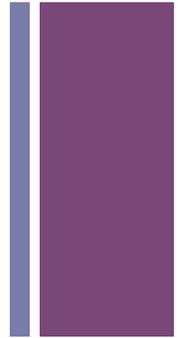
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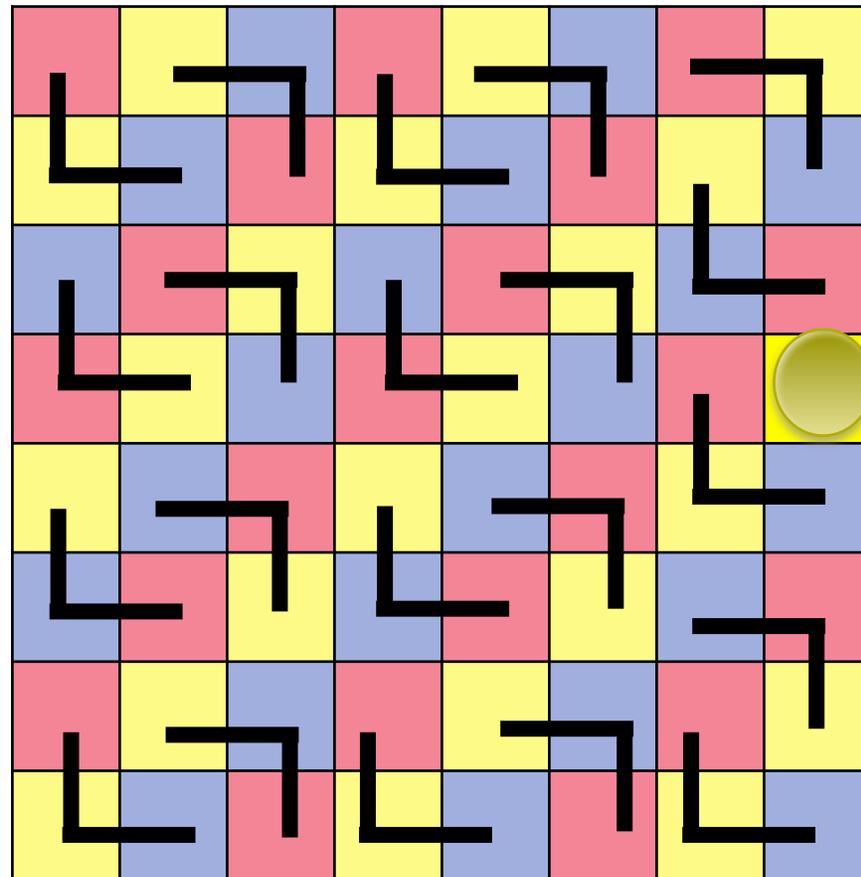
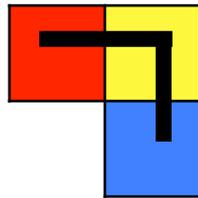
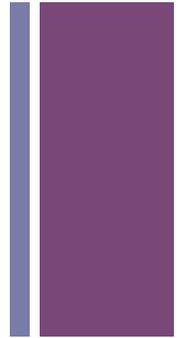
+ Q4: Is it possible to tile an 8x8 board with 21 “L-shaped” tiles of three squares and one hole?



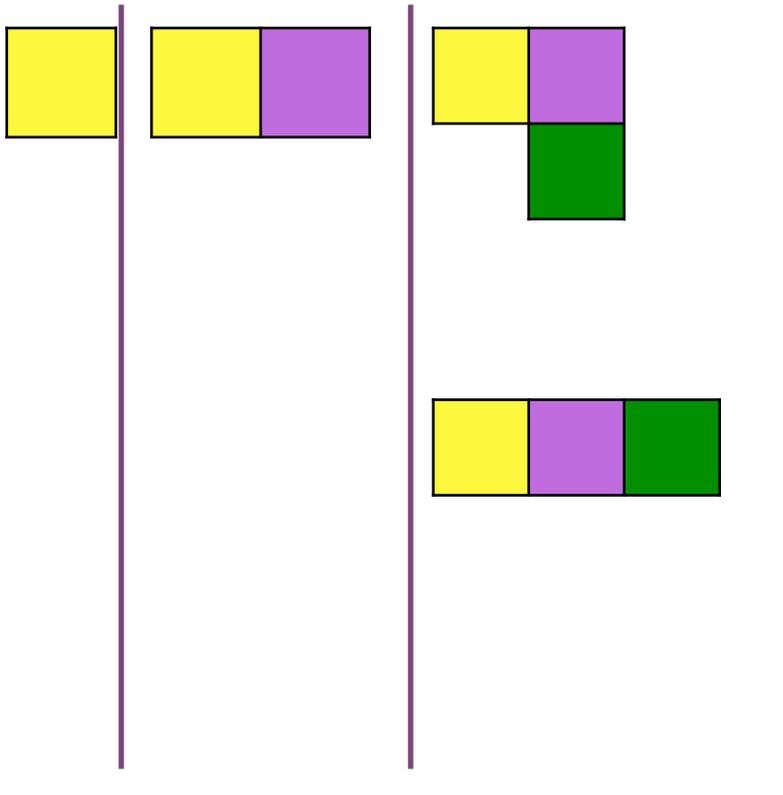
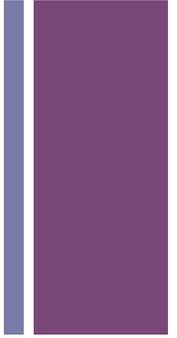
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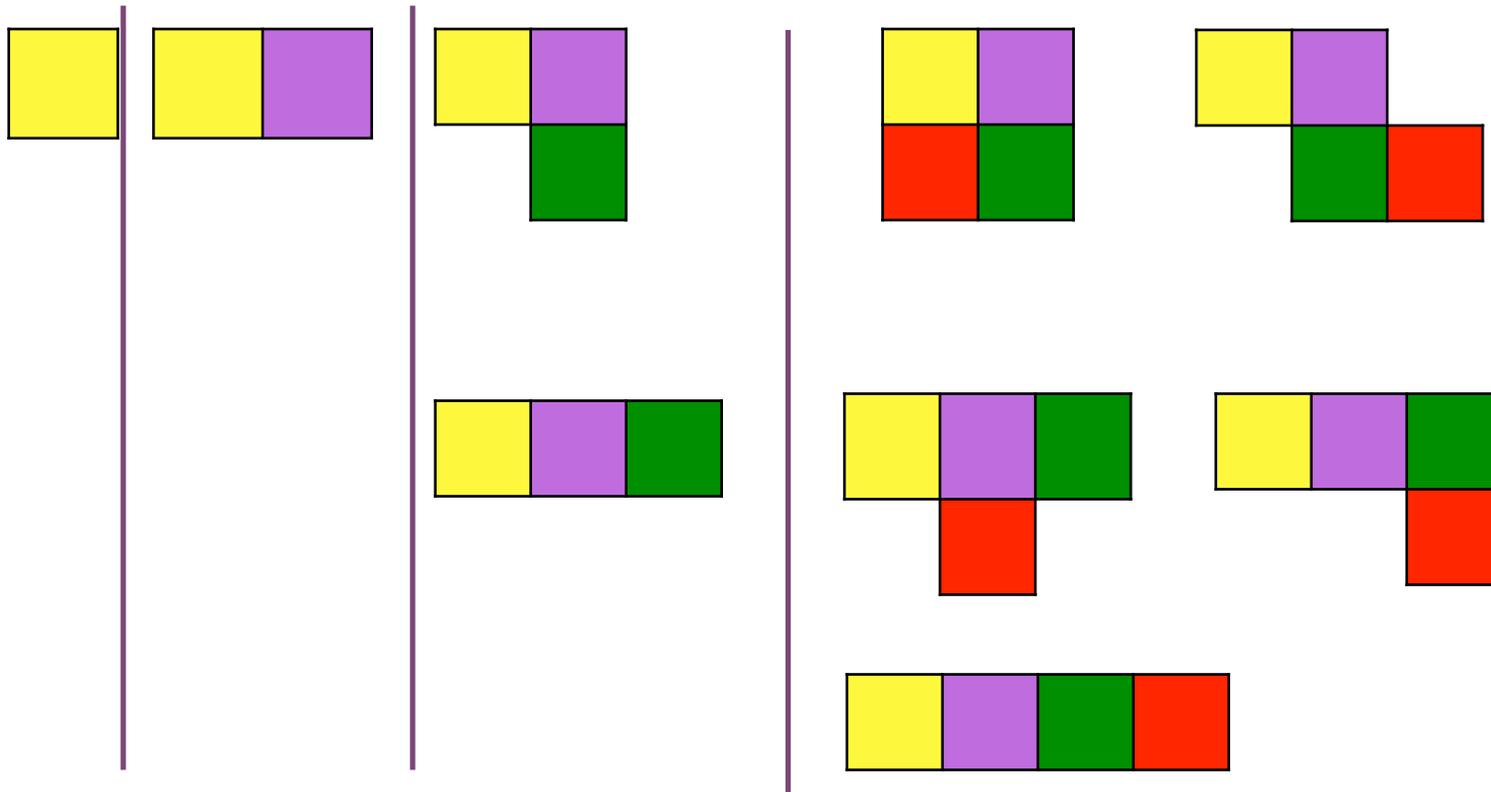
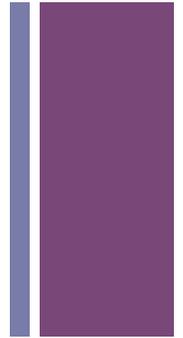
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+ Other Questions to think about...
What other polyominoes are there?



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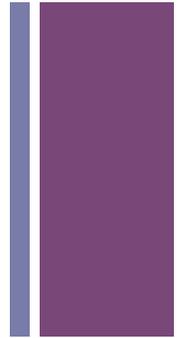
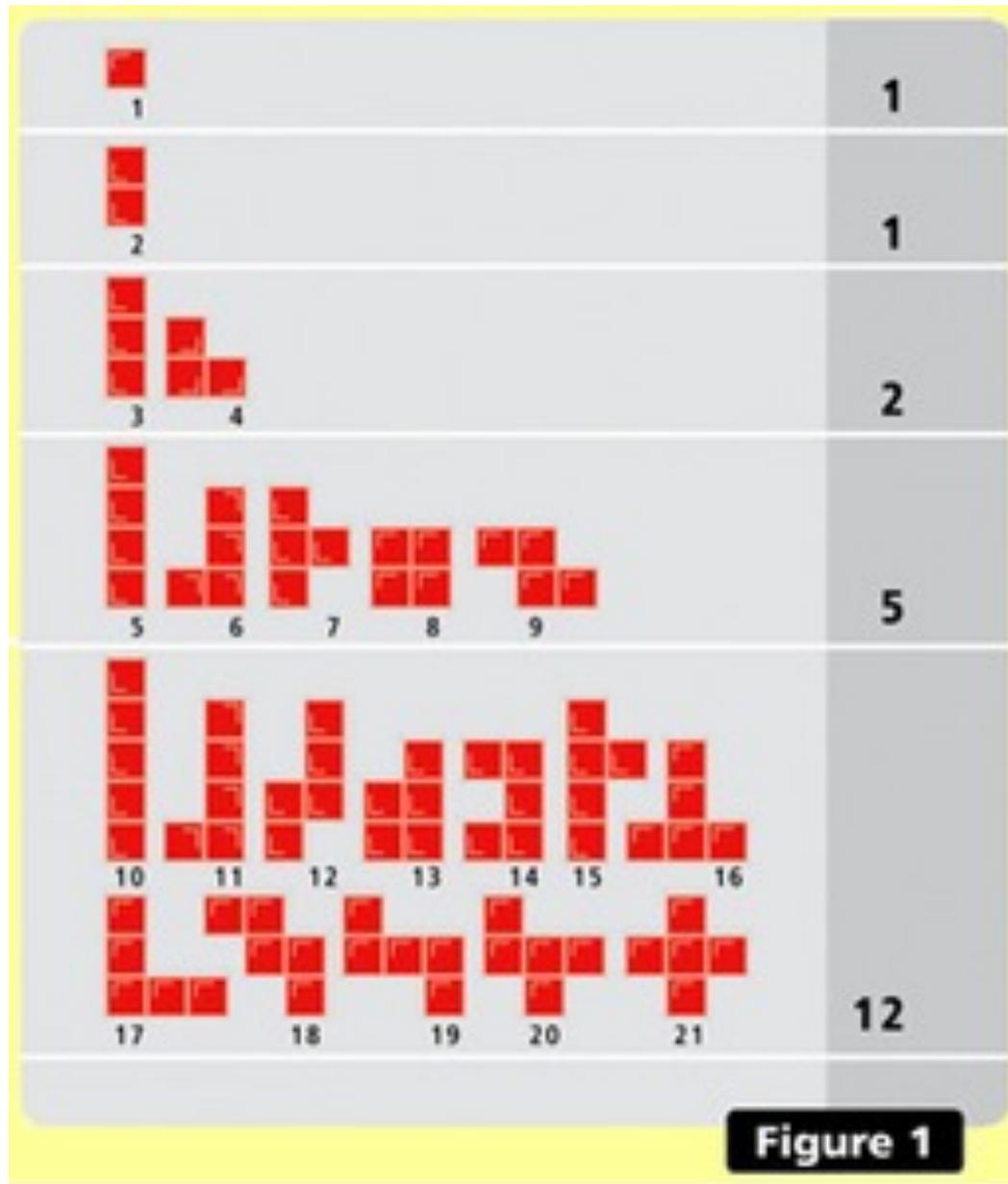
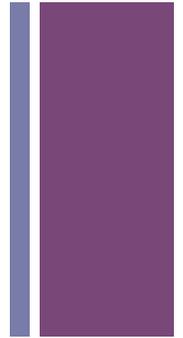


Image from: <http://seshil4324jp.sugo-roku.com/newgame/blokus>

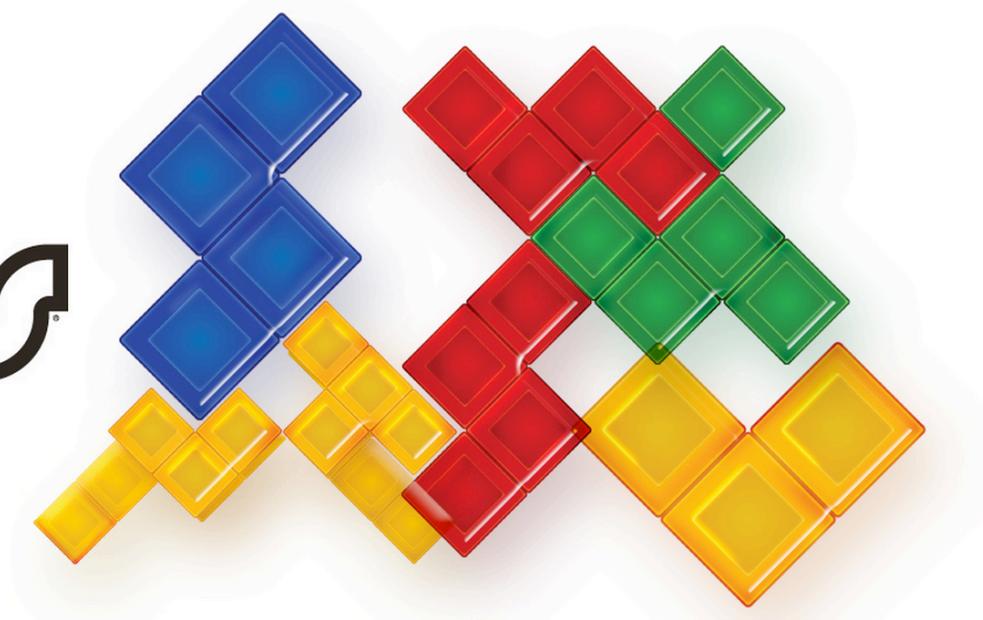


Blokus!

<http://mattelgames.com/en-us/blokus>

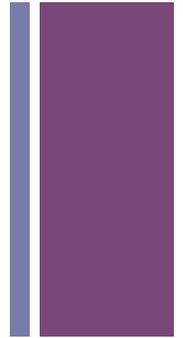
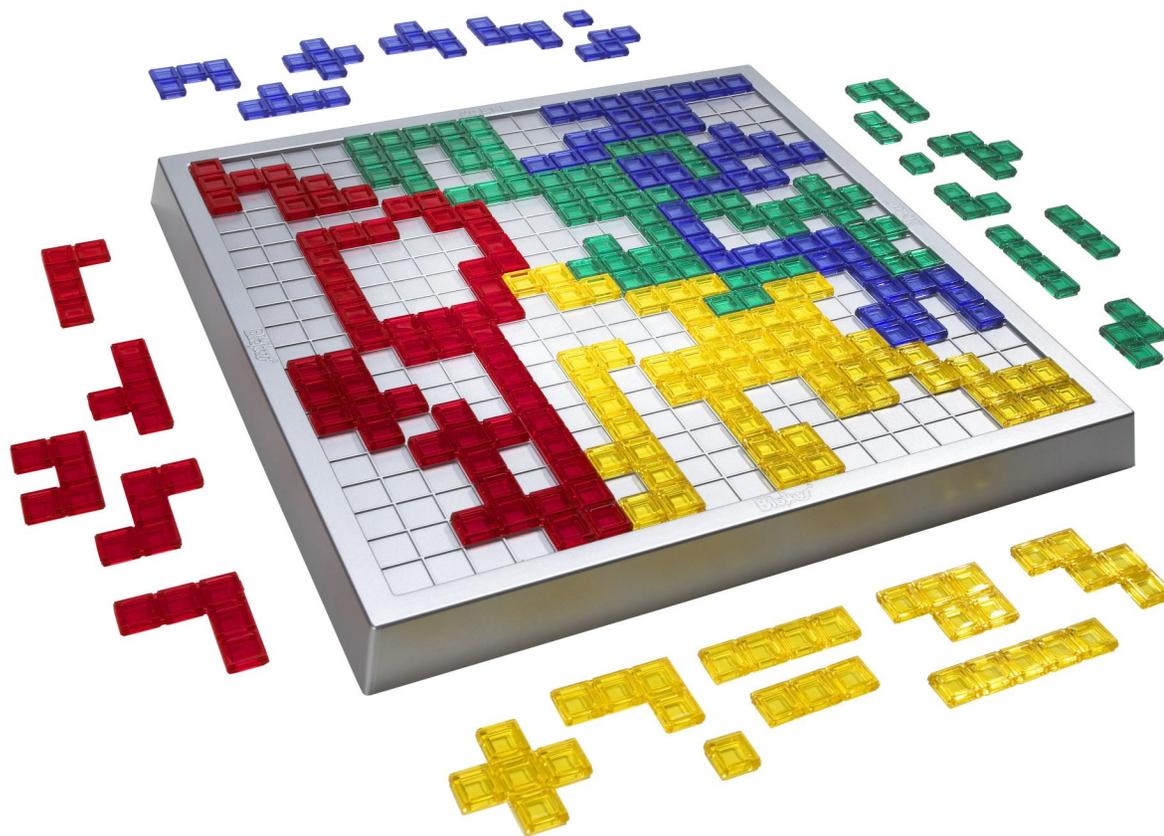


Blokus[®]

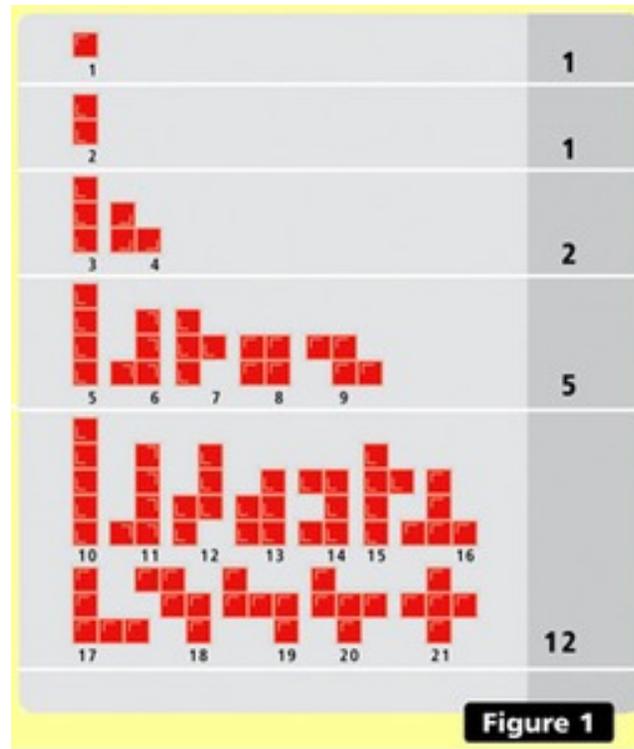
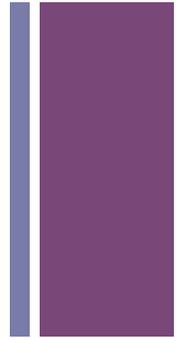


+ Blokus!

<http://mattelgames.com/en-us/blokus>



+ Other Questions to think about...
Can you tile all the 1 through 5
polyominoes into 1 rectangle?



+ Polyominoes of Order 1 through 6

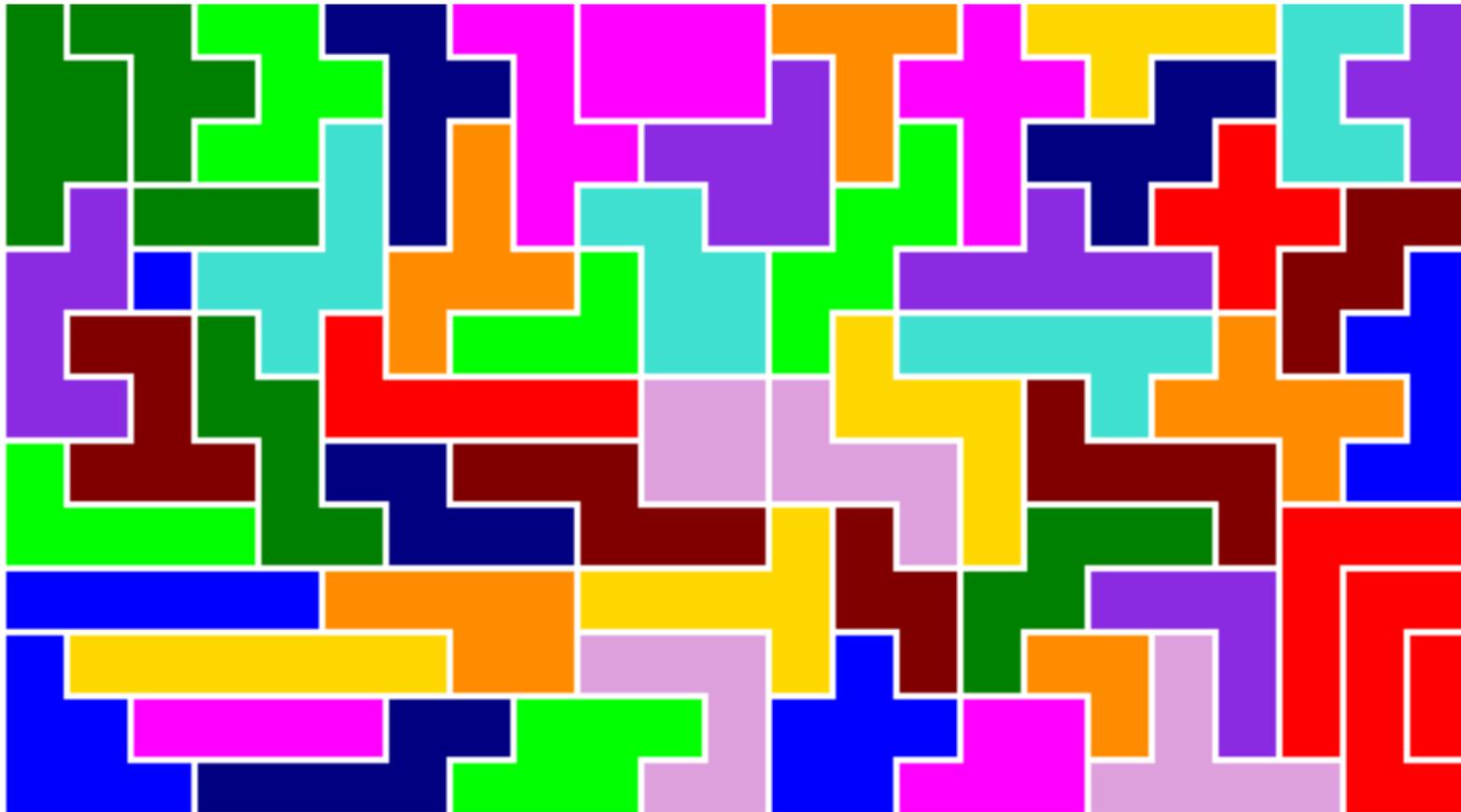


Image from <http://puzzler.sourceforge.net/docs/polyominoes.html>



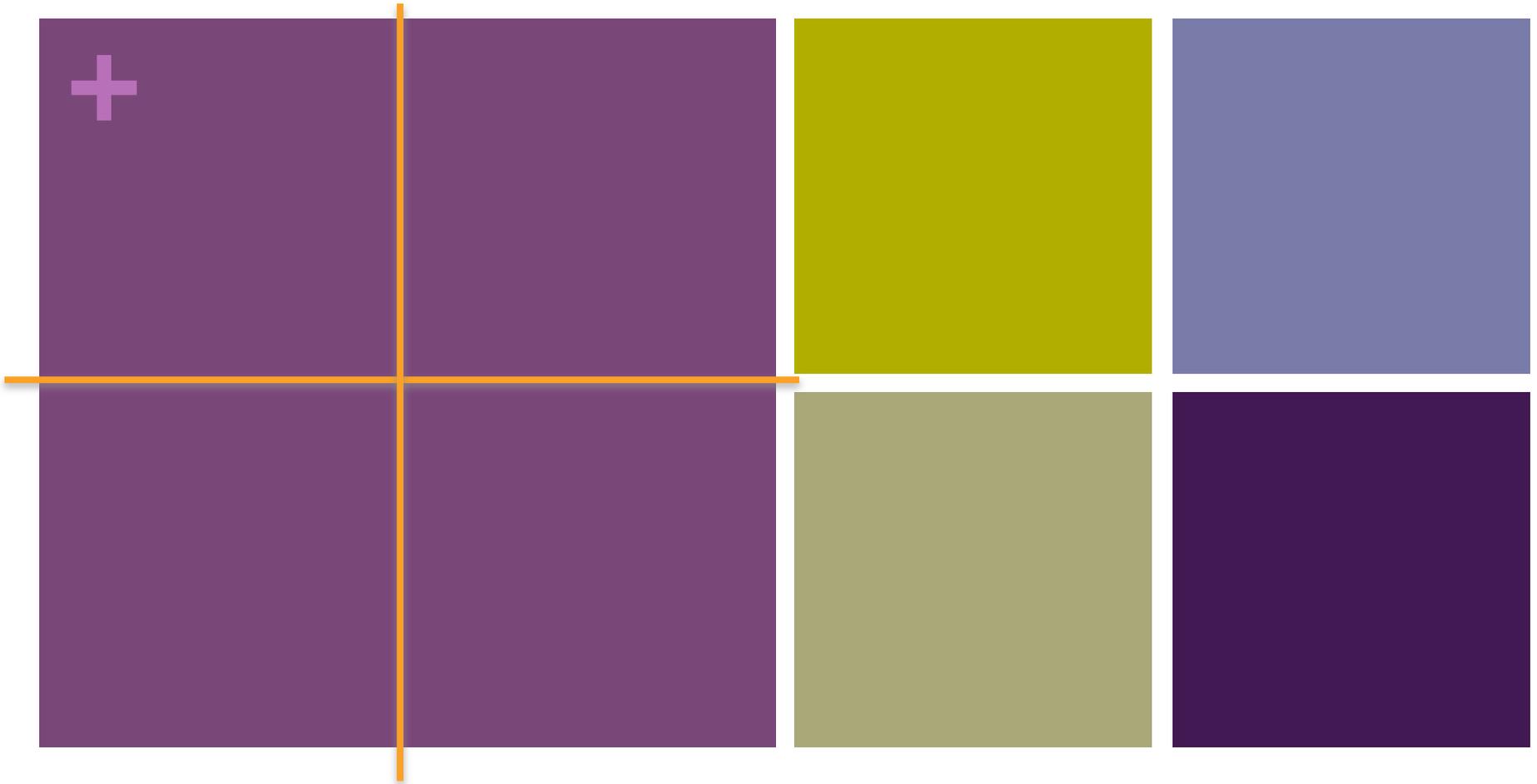
Other Questions to think about...
What if your tiles have to be squares of 2^n length?
What shapes can you tile then?

Mathematical Tiling and Organization



Dr. Brandy Wieggers
Central Washington University





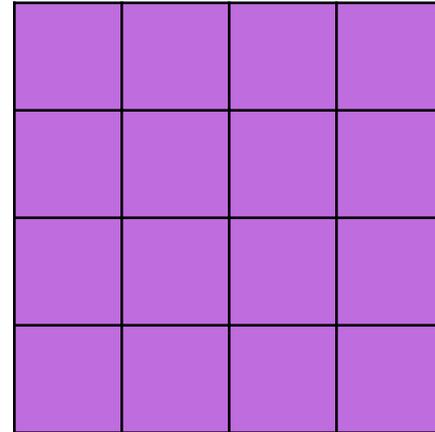
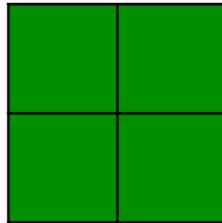
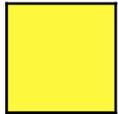
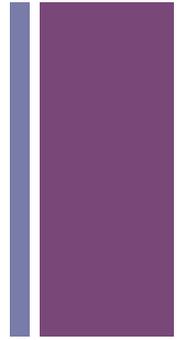
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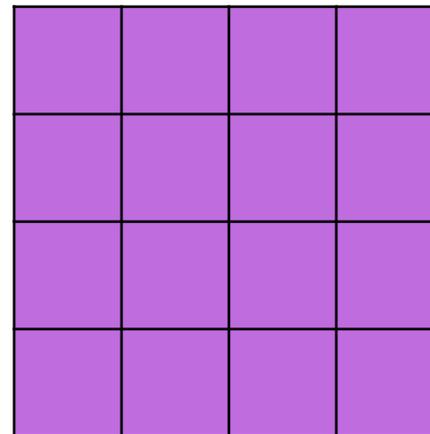
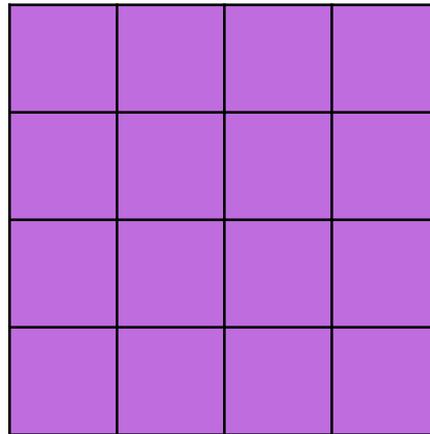
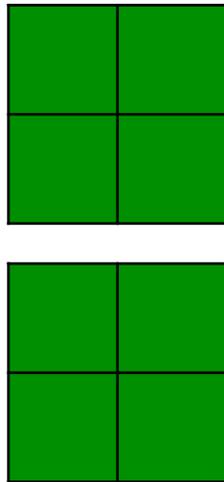
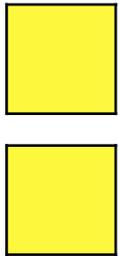
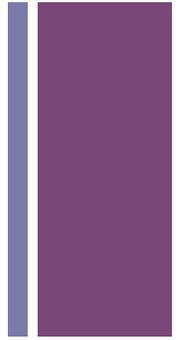
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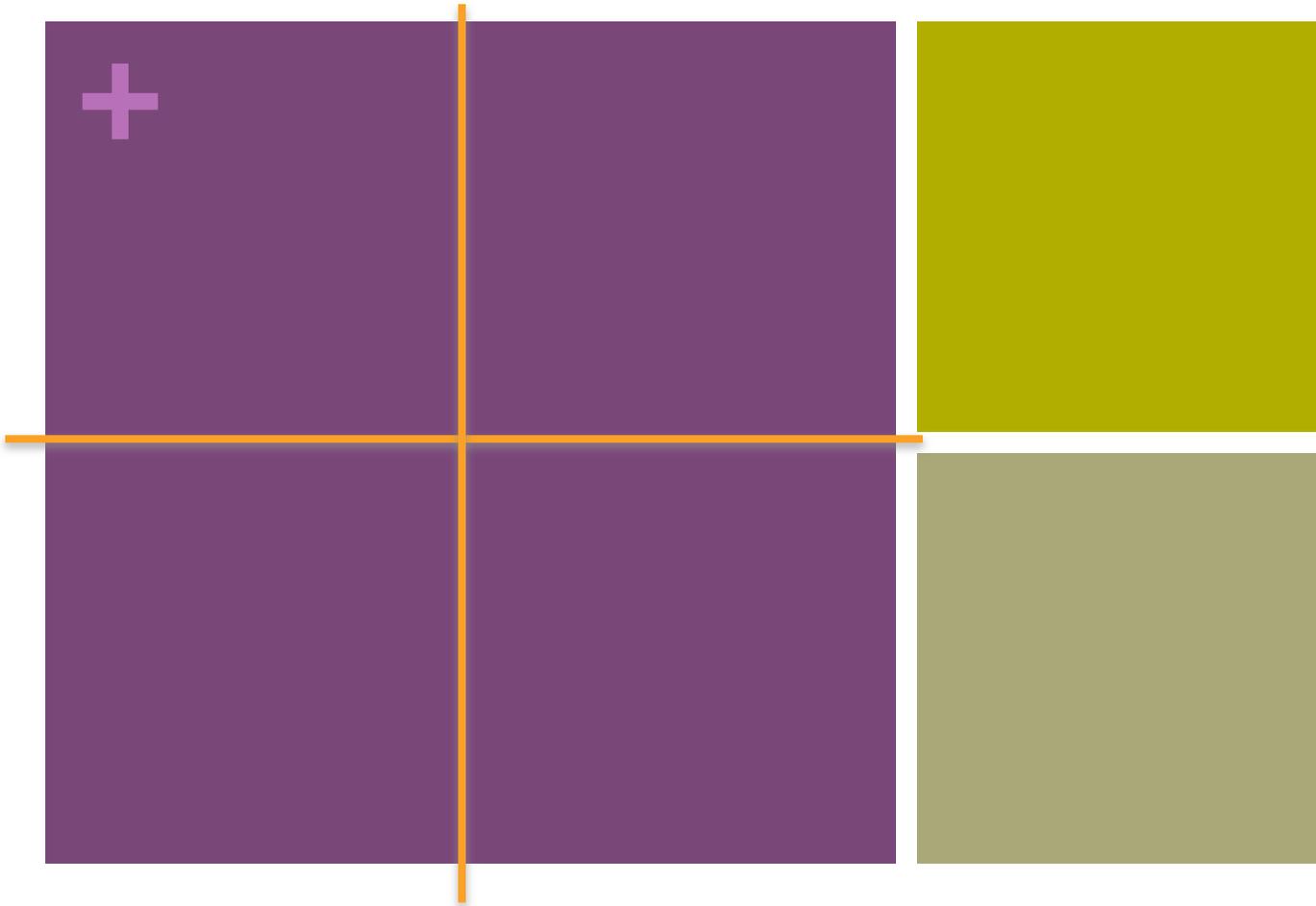
Q: What if your tiles have to be squares of 2^n length? What shapes can you tile if you can only use 1 tile of each length?



+

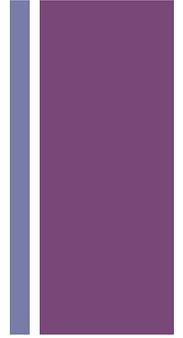
Q: What if your tiles have to be squares of 2^n length? What shapes can you tile if you can only use 2 tiles of each length?





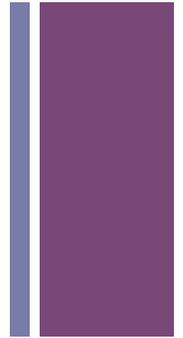
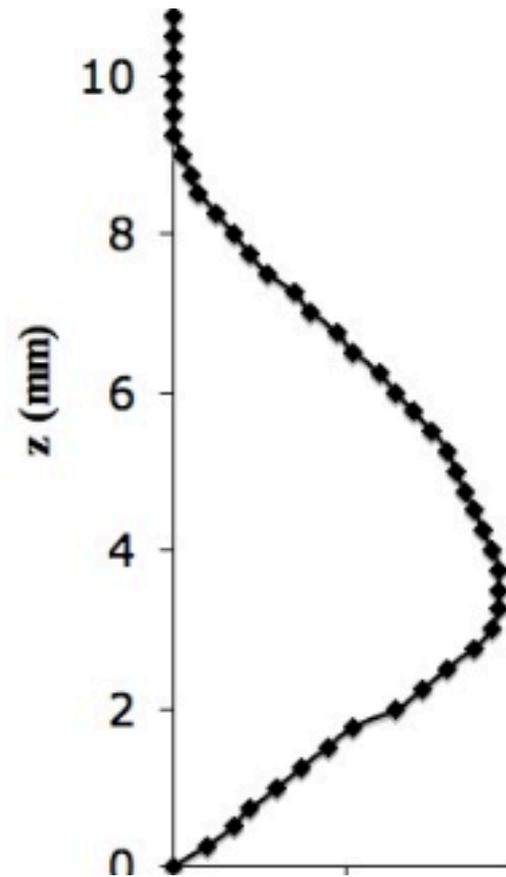
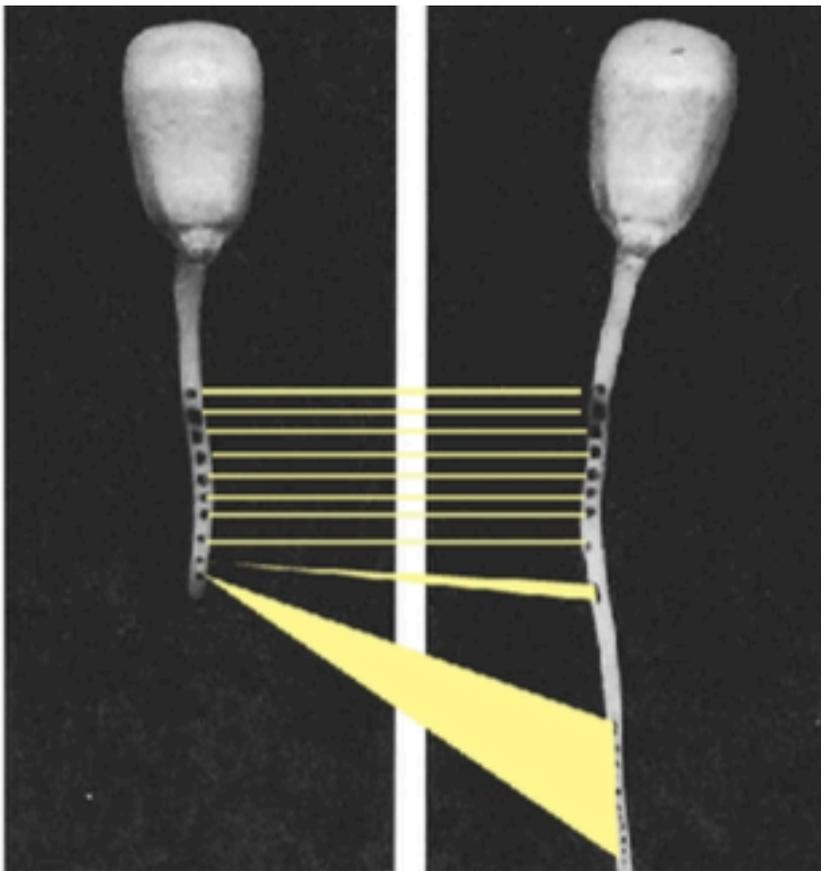
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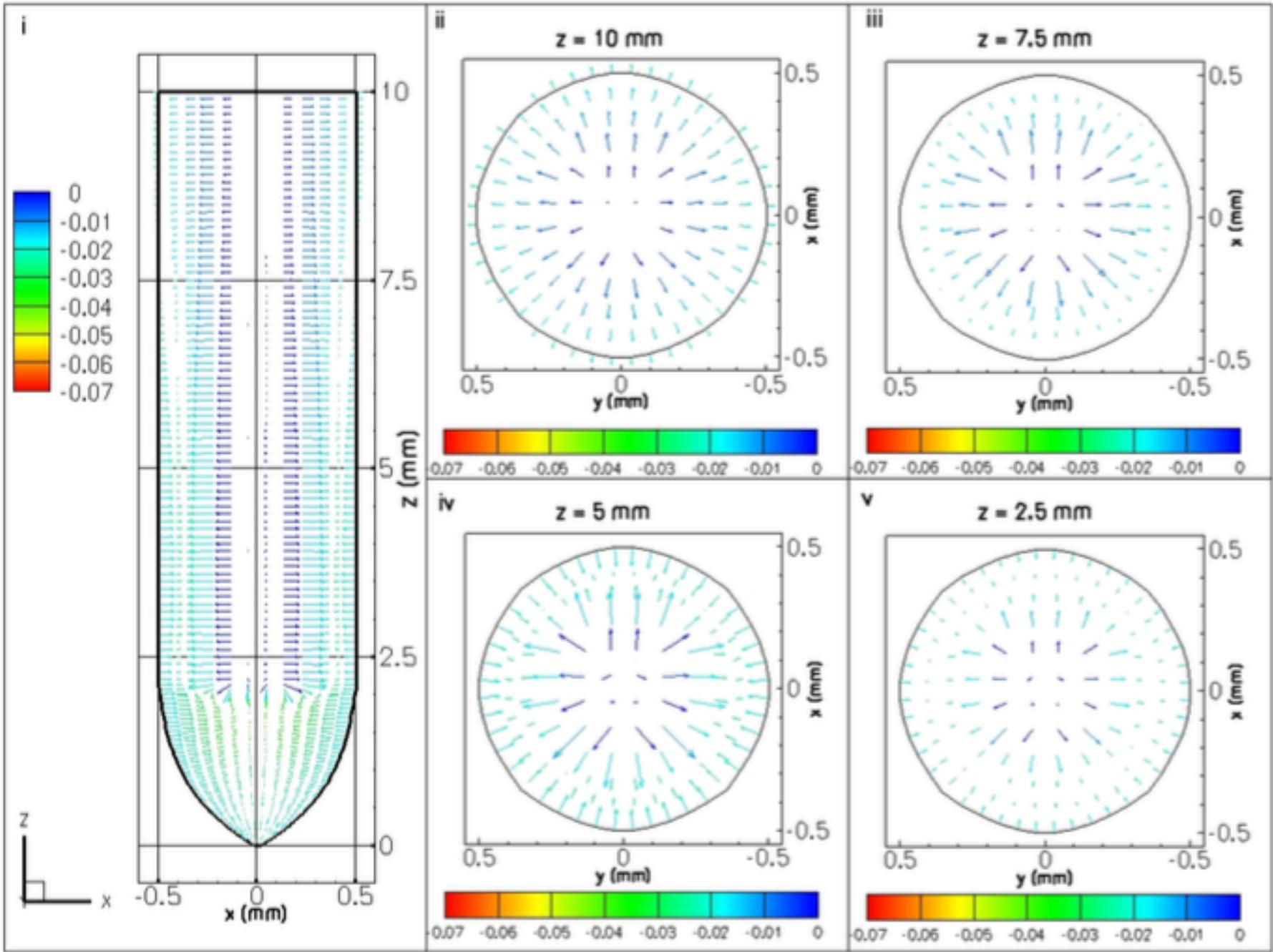
+ Problem Solving Skills

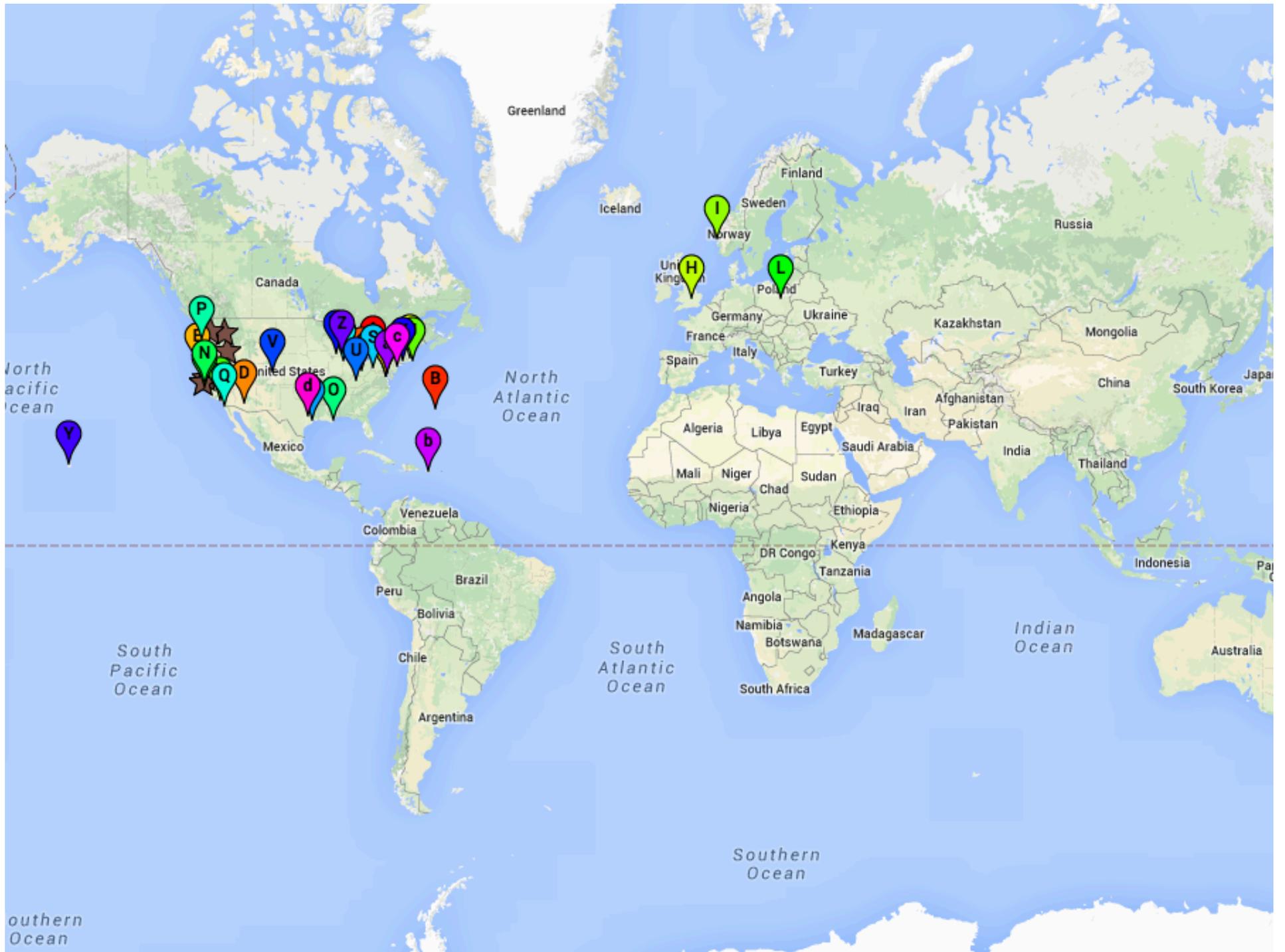


- Mathematical Play
- Using Organization!
 - Starting smaller
 - Recording Data
 - Designing systems to find repeats
 - Coloring

+ My Ph.D.

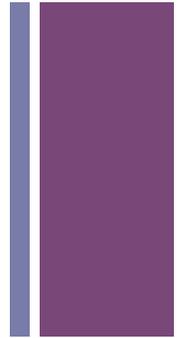








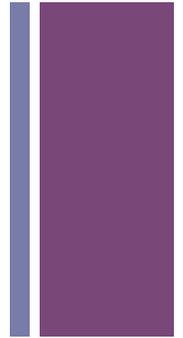
+ Julia Robinson Math Festival
– April 4th (1-5pm), UW HUB



Julia Robinson
Mathematics Festival

<http://mathforlove.com/julia-robinson-festival/>

+ Any questions?



Dr. Brandy Wiegers

wiegersb@cwu.edu

