The Mathemagic of Magic Squares

Steven Klee

University of California, Davis

April 15, 2012
Warm-Up

The 15 Game

Players take turns choosing numbers between 1 and 9, without repeats. The first player to choose 3 numbers that add up to 15 wins.
Warm-Up

The 15 Game

Players take turns choosing numbers between 1 and 9, without repeats. The first player to choose 3 numbers that add up to 15 wins.

1 2 3 4 5 6 7 8 9

Player 1: 3, 6, 8, 4
Player 2: 2, 5, 1
Warm-Up

The 15 Game

Players take turns choosing numbers between 1 and 9, without repeats. The first player to choose 3 numbers that add up to 15 wins.

1 2 3 4 5 6 7 8 9

Player 1: 3

Player 2:
Warm-Up

The 15 Game

Players take turns choosing numbers between 1 and 9, without repeats. The first player to choose 3 numbers that add up to 15 wins.

1  4  5  6  7  8  9

Player 1: 3
Player 2: 2
Warm-Up

The 15 Game

Players take turns choosing numbers between 1 and 9, without repeats. The first player to choose 3 numbers that add up to 15 wins.

1  4  5  7  8  9

Player 1: 3, 6

Player 2: 2
Warm-Up

The 15 Game

Players take turns choosing numbers between 1 and 9, without repeats. The first player to choose 3 numbers that add up to 15 wins.

1  4  7  8  9

Player 1: 3, 6

Player 2: 2, 5
Warm-Up

The 15 Game

Players take turns choosing numbers between 1 and 9, without repeats. The first player to choose 3 numbers that add up to 15 wins.

1 2 3 4 5 6 7 8 9

Player 1: 3, 6, 8
Player 2: 2, 5
Warm-Up

The 15 Game

Players take turns choosing numbers between 1 and 9, without repeats. The first player to choose 3 numbers that add up to 15 wins.

4 7 9

Player 1: 3, 6, 8
Player 2: 2, 5, 1
Warm-Up

The 15 Game

Players take turns choosing numbers between 1 and 9, without repeats. The first player to choose 3 numbers that add up to 15 wins.

1 2 3 4 5 6 7 8 9

Player 1: 3, 6, 8, 4
Player 2: 2, 5, 1
Warm-Up

The 15 Game

Players take turns choosing numbers between 1 and 9, without repeats. The first player to choose 3 numbers that add up to 15 wins.

7
9

Player 1:
3, 6, 8, 4

Player 2:
2, 5, 1
1. What is a Magic Square?

2. History of Magic Squares

3. Mathematics and Magic Squares

4. Constructing Magic Squares

5. Magic Circles
Definition

A **magic square** is a filling of an $n \times n$ square with the numbers $1, 2, \ldots, n^2$ so that the rows, columns, and diagonals all sum to the same number.
Definition

A **magic square** is a filling of an $n \times n$ square with the numbers $1, 2, \ldots, n^2$ so that the rows, columns, and diagonals all sum to the same number.
The Lo Shu Square

**Lo Shu Square:** $\sim 650$ BCE

Magic Sum 15 is the number of days in the 24 cycles of the Chinese solar year.
The Chautisa Yantra

Chautisa Yantra: Parshvanath Jain temple in Khajuraho, India (10th century)
Dürer’s Square

Albrecht Dürer: Melencolia I (1514)
Benjamin Franklin’s Squares

“The Governor put me into the commission of the Peace; the Corporation of the City chose me of the Common Council, and soon after an Alderman; and the Citizens at large chose me a Burgess to represent them in Assembly. This latter Station was the more agreeable to me, as I was at length tired with sitting there to hear Debates in which as Clerk I could take no part, and which were often so unentertaining, that I was induced to amuse myself with making magic squares, or circles, or anything to avoid weariness.”
### Benjamin Franklin’s Magic Square

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**Question:** What is the magic sum for an $n \times n$ magic square?

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So

\[
n \cdot S = 1 + 2 + 3 + \cdots + n^2
\]
**Question:** What is the magic sum for an $n \times n$ magic square?

So

$$n \cdot S = 1 + 2 + 3 + \cdots + n^2$$

$$= \frac{n^2(n^2 + 1)}{2}$$
The Magic Sum

**Question:** What is the magic sum for an $n \times n$ magic square?

\[
\begin{array}{cccccc}
\end{array}
\]

\[n \cdot S = \frac{n^2(n^2 + 1)}{2} \]

So

\[n \cdot S = 1 + 2 + 3 + \cdots + n^2 = \frac{n^2(n^2 + 1)}{2}\]

\[S = \frac{n(n^2 + 1)}{2}\]
The Magic Sum

The magic sum for an $n \times n$ magic square is

$$\frac{n(n^2 + 1)}{2}.$$

**Example:**

- $n = 3$:
  $$S = \frac{3 \cdot (3^2 + 1)}{2} = \frac{3 \cdot 10}{2} = 15$$

- $n = 4$:
  $$S = \frac{4 \cdot (4^2 + 1)}{2} = \frac{4 \cdot 17}{2} = 34$$

- $n = 5$:
  $$S = \frac{5 \cdot (5^2 + 1)}{2} = \frac{5 \cdot 26}{2} = 65$$

- $n = 8$:
  $$S = \frac{8 \cdot (8^2 + 1)}{2} = \frac{8 \cdot 65}{2} = 260$$
The 15 game

Rules

Two players take turns choosing numbers between 1 and 9. The objective is to collect three numbers that sum to 15.
The 15 game

Rules

Two players take turns choosing numbers between 1 and 9. The objective is to collect three numbers that sum to 15.

Winning collections:

1 + 9 + 5
1 + 8 + 6
2 + 9 + 4
2 + 8 + 5
2 + 7 + 6
3 + 8 + 4
3 + 7 + 5
4 + 6 + 5
The 15 game

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Two players take turns choosing numbers between 1 and 9. The objective is to collect three numbers that sum to 15.

Winning collections:

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Two players take turns choosing numbers between 1 and 9. The objective is to collect three numbers that sum to 15.

Winning collections:

- $1 + 9 + 5$
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1 + 9 + 5
1 + 8 + 6
2 + 9 + 4
2 + 8 + 5
2 + 7 + 6
3 + 8 + 4
3 + 7 + 5
4 + 6 + 5

8 1 6
 5 7
 2
The 15 game

Rules

Two players take turns choosing numbers between 1 and 9. The objective is to collect three numbers that sum to 15.

Winning collections:

1 + 9 + 5
1 + 8 + 6
2 + 9 + 4
2 + 8 + 5
2 + 7 + 6
3 + 8 + 4
3 + 7 + 5
4 + 6 + 5
The 15 game

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4 + 6 + 5

8 1 6
3 5 7
4 9 2
The 15 game

**Rules**

Two players take turns choosing numbers between 1 and 9. The objective is to collect three numbers that sum to 15.

**Winning collections:**

**Player 1:**

```
 8 1 6
3 5 7
4 9 2
```

**Player 2:**

```
8 1 6
3 5 7
4 9 2
```
The 15 game

Rules

Two players take turns choosing numbers between 1 and 9. The objective is to collect three numbers that sum to 15.

Winning collections:

Player 1:

3

Player 2:

\[
\begin{array}{ccc}
8 & 1 & 6 \\
3 & 5 & 7 \\
4 & 9 & 2 \\
\end{array}
\]
The 15 game

Rules

Two players take turns choosing numbers between 1 and 9. The objective is to collect three numbers that sum to 15.

Winning collections:

Player 1:

3

Player 2:

\[
\begin{array}{ccc}
8 & 1 & 6 \\
X & 5 & 7 \\
4 & 9 & 2 \\
\end{array}
\]
The 15 game

Rules

Two players take turns choosing numbers between 1 and 9. The objective is to collect three numbers that sum to 15.

Winning collections:

Player 1:

3

\[
\begin{array}{ccc}
8 & 1 & 6 \\
\times & 5 & 7 \\
4 & 9 & 2 \\
\end{array}
\]

Player 2:

2
The 15 game

Rules

Two players take turns choosing numbers between 1 and 9. The objective is to collect three numbers that sum to 15.

Winning collections:

Player 1:

3

Player 2:

2

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</table>
The 15 game

Rules

Two players take turns choosing numbers between 1 and 9. The objective is to collect three numbers that sum to 15.

Winning collections:

Player 1:

3, 6

Player 2:

2

Winning collections:

Player 1:

3, 6

Player 2:

2
The 15 game

Rules

Two players take turns choosing numbers between 1 and 9. The objective is to collect three numbers that sum to 15.

Winning collections:

Player 1:
3, 6

Player 2:
2

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Player 1:

3, 6

Player 2:

2, 5
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Winning collections:

Player 1:

3, 6

Player 2:

2, 5
The 15 game

Rules

Two players take turns choosing numbers between 1 and 9. The objective is to collect three numbers that sum to 15.

Winning collections:

Player 1:

3, 6, 8

Player 2:

2, 5

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<td>X</td>
<td>O</td>
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</table>

| 4 | 9 | O |
The 15 game

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Winning collections:

Player 1:
3, 6, 8

Player 2:
2, 5, 1
The 15 game

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Winning collections:

Player 1:
3, 6, 8, 4

Player 2:
2, 5, 1
Constructing Odd Magic Squares

1. Place 1 in the middle of the top row.
2. Having placed number $i$, place number $i + 1$:
   1. One square to the northeast of $i$, if you can (wrapping if necessary).
   2. One square to the south of $i$, otherwise.

\[
\begin{array}{|c|c|c|}
\hline
1 & 6 & 3 \\
\hline
8 & 1 & 4 \\
\hline
3 & 5 & 7 \\
\hline
\end{array}
\]
Constructing Odd Magic Squares

1. Place 1 in the middle of the top row.
2. Having placed number \( i \), place number \( i + 1 \):
   1. One square to the northeast of \( i \), if you can (wrapping if necessary).
   2. One square to the south of \( i \), otherwise.

\[
\begin{array}{ccc}
& & \\
& 1 & \\
& & \\
\end{array}
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 & & \\
 & 1 & \\
 & & \\
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\[
\begin{array}{ccc}
1 & & \\
 & & \\
 & \rightarrow & 2
\end{array}
\]
Constructing Odd Magic Squares

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1 & & \\
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\[
\begin{array}{ccc}
1 & & \\
3 & & \\
4 & 2 & \\
\end{array}
\]
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\[
\begin{array}{ccc}
1 & & \\
3 & 5 & \\
4 & 2 & \\
\end{array}
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\[
\begin{array}{ccc}
1 & 6 \\
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\[
\begin{array}{ccc}
1 & 6 & \\
3 & 5 & 7 \\
4 & 2 & \\
\end{array}
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Constructing Odd Magic Squares

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\[
\begin{array}{ccc}
8 & 1 & 6 \\
3 & 5 & 7 \\
4 & 2 & \\
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```
  8  1  6
  3  5  7
  4  9  2
```
Constructing Odd Magic Squares

1. Place 1 in the middle of the top row.
2. Having placed number $i$, place number $i + 1$:
   1. One square to the northeast of $i$, if you can (wrapping if necessary).
   2. One square to the south of $i$, otherwise.

$$
\begin{array}{cccccc}
17 & 24 & 1 & 8 & 15 \\
23 & 5 & 7 & 14 & 16 \\
4 & 6 & 13 & 20 & 22 \\
10 & 12 & 19 & 21 & 3 \\
11 & 18 & 25 & 2 & 9 \\
\end{array}
$$
What about even Magic Squares?

When \( n = 2 \cdot (2m + 1) \)

1. Start with a \( 2m + 1 \times 2m + 1 \) magic square.
2. Fill another \( 2m + 1 \times 2m + 1 \) square with the letters L, U, and X as follows:
What about even Magic Squares?

When \( n = 2 \cdot (2m + 1) \)

1. Start with a \( 2m + 1 \times 2m + 1 \) magic square.
2. Fill another \( 2m + 1 \times 2m + 1 \) square with the letters L, U, and X as follows:
   1. Fill the first \( m + 1 \) rows with L.
   2. Fill the next row with U.
   3. Fill the remaining rows with X.
   4. Replace the middle entry of the U row with the L above it.

\[
\begin{array}{ccc}
8 & 1 & 6 \\
3 & 5 & 7 \\
4 & 9 & 2 \\
\end{array}
\]

\[
\begin{array}{ccc}
L & L & L \\
L & U & L \\
U & L & U \\
\end{array}
\]
3. Replace each square in the LUX grid with a $2 \times 2$ square according to the rules:

\[
\begin{array}{ccc}
4 & 1 & 1 \\
2 & 3 & 3 \\
\end{array}
\begin{array}{ccc}
1 & 4 & 2 \\
2 & 3 & 3 \\
\end{array}
\begin{array}{ccc}
1 & 4 & 3 \\
\end{array}
\]
The LUX Method

The Mathemagic of Magic Squares

Steven Klee

Outline

What is a Magic Square?
History of Magic Squares
Mathematics and Magic Squares
Constructing Magic Squares
Magic Circles

The LUX Method

2 4
1 3
2 3
1 4
3 2

LUX Method:

1 6
3 5 7
4 9 2
L L L
L U L
U L U

32 29 4 1 24 21
30 31 2 3 22 23
12 9 17 20 28 25
10 11 18 19 26 27
13 16 36 33 5 8
14 15 34 35 6 7
The LUX Method

**The Mathemagic of Magic Squares**

Steven Klee

Outline

What is a Magic Square?

History of Magic Squares

Mathematics and Magic Squares

Constructing Magic Squares

Magic Circles

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The LUX Method

![Diagram of LUX method](image)

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The Mathemagic of Magic Squares

Steven Klee

Outline
What is a Magic Square?
History of Magic Squares
Mathematics and Magic Squares
Constructing Magic Squares
Magic Circles

The LUX Method

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Ben Franklin’s Magic Circles

“Dear Sir, As you seemed desirous of seeing the magic circle I mentioned to you, I have revised the one I made many years since, and with some improvements, sent it to you.” In a letter to John Canton, May 29, 1765.
Benjamin Franklin’s Magic Circle
The Mathemagic of Magic Squares

Steven Klee

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Radial Sum
Outer-half Radial Sum

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Magic Circles
Inner-half Radial Sum
The Mathemagic of Magic Squares
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Annular Sum
Upper-half Annular Sum
2×2 Block Sums

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Outline

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Magic Circles

Northern Excentric Annular Sum
Eastern Excentric Annular Sum
Southern Excentric Annular Sum
Western Excentric Annular Sum
Vertically-centered Excentric Lower Half-annular Sum
Vertically-centered Excentric Upper Half-annular Sum
Horizontally-centered Excentric Right Half-annular Sum
Horizontally-centered Excentric Left Half-annular Sum
“The magic square and circle, I am told, have occasioned a good deal of puzzling among the mathematicians here, but no one has desired me to show him my method of disposing the numbers. It seems they wish rather to investigate it themselves.” In a letter to John Winthrop, July 2, 1768
Thank you!