University of Washington Math Hour Olympiad, 2024

Grades 8–10

Problem #1 Arturo needs to move six paintings by different artists between floors in a museum:

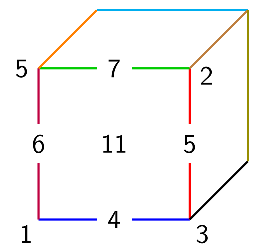
Paul's painting:	Floor 4 to Floor 9	Yayoi's painting:	Floor 2 to Floor 7
Frida's painting:	Floor 4 to Floor 1	Leonardo's painting:	Floor 7 to Floor 4
Rene's painting:	Floor 2 to Floor 4	Georgia's painting:	Floor 8 to Floor 2

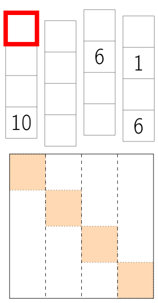
The paintings can be moved in any order, but the elevator can only hold one painting at a time. Arturo starts and ends on Floor 1. What is the smallest number of floors he can travel and deliver all the paintings to their desired destinations? (For example, if he takes the elevator from Floor 1 to Floor 7, then from Floor 7 to Floor 3, he has traveled $(7-1)+(7-3)=10$ floors.)

Problem #2 Harry has a magic cube. To open it, he must write integers on each of the 8 corners, 12 edges, and 6 faces. The cube will unlock if

- the number on each edge is the sum of the two corners it connects,
- the number on each face is the sum of the four corners around it, and
- the sum of all $8+12+6=26$ numbers is 2024.

Can Harry unlock the cube?





Problem #3 Martine starts with 4 strips of paper that have 4 filled cells, as shown. She fills in the 12 empty cells with integers.

Whenever she arranges the strips in the square board to the left (possibly changing their order, but not rotating/flipping them), the cells on the shaded diagonal always add to 30.

Prove that the cell outlined in red does not contain the largest of the 16 integers.

Problem #4 A group of children were playing "king of the tennis court." To start, one child is named king, and the rest form a line. The first player in line challenges the king to a tennis game. The winner becomes the new king, the loser goes to the back of the line, and the process repeats.

The group played a total of 75 games. Naomi won 12 games and lost 8. Prove that Naomi was not the king after the last game.

Problem #5 There are 21 cards, each labeled with a positive integer. The cards are arranged face up in 10 piles of two and 1 pile of one. All 21 card numbers are always visible.

Alice and Bob alternate turns picking cards. On their turn, a player must take one card that is currently on top of its pile. Once all 21 cards are taken, the player whose cards have the larger sum wins.

Alice takes the first card. Prove that Alice can always win.