University of Washington Math Hour Olympiad, 2024

Grades 6-7

Problem #6 A group of children were playing "king of the tennis court." To start, one child is named king, and the rest form a line. The first player in line challenges the king to a tennis game. The winner becomes the new king, the loser goes to the back of the line, and the process repeats.

The group played a total of 75 games. Naomi won 12 games and lost 8. Prove that Naomi was not the king after the last game.



Problem #7 There are 21 cards, each labeled with a positive integer. The cards are arranged face up in 10 piles of two and 1 pile of one. All 21 card numbers are always visible.

Alice and Bob alternate turns picking cards. On their turn, a player must take one card that is currently on top of its pile. Once all 21 cards are taken, the player whose cards have the larger sum wins.

Alice takes the first card. Prove that Alice can always win.

