

University of Washington Math Hour Olympiad 2023

Grades 6–7



Problem #1 Ash is running around town catching Pokémon. Each day, he may add 3, 4, or 5 Pokémon to his collection, but he can never add the same number of Pokémon on two consecutive days. What is the smallest number of days it could take for him to collect exactly 100 Pokémon?

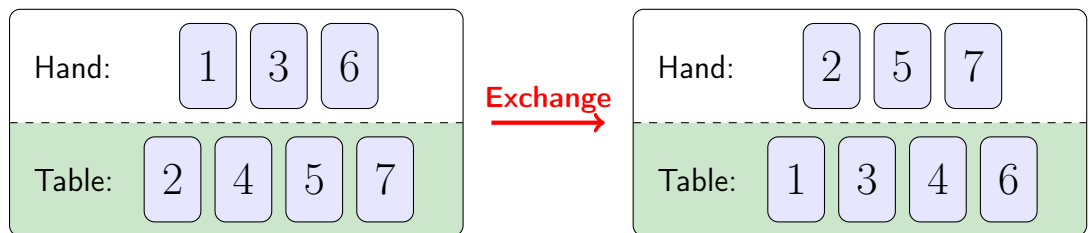
Problem #2 Jack and Jill have ten buckets. One bucket can hold up to 1 gallon of water, another can hold up to 2 gallons, and so on, with the largest able to hold up to 10 gallons.

The ten buckets are arranged in a line as shown below. Jack and Jill can pour some amount of water into each bucket, but no bucket can have less water than the one to its left. Is it possible that together, the ten buckets can hold 36 gallons of water?



Problem #3 There are 2023 knights and liars standing in a row. Knights always tell the truth and liars always lie. Each of them says, “the number of liars to the left of me is greater than the number of knights to the right.” How many liars are there?

Problem #4 Camila has a deck of 101 cards numbered 1, 2, ..., 101. She starts with 50 random cards in her hand and the rest on a table with the numbers visible. In an *exchange*, she replaces all 50 cards in her hand with her choice of 50 of the 51 cards from the table. Show that Camila can make at most 50 exchanges and end up with cards 1, 2, ..., 50.



Problem #5 There are 101 pirates on a pirate ship: the captain and 100 crew. Each pirate, including the captain, starts with 1 gold coin. The captain makes proposals for redistributing the coins, and the crew vote on these proposals. The captain does not vote.

For every proposal, each crew member greedily votes “yes” if he gains coins as a result of the proposal, “no” if he loses coins, and passes otherwise. If strictly more crew members vote “yes” than “no,” the proposal takes effect.

The captain can make any number of proposals, one after the other. What is the largest number of coins the captain can accumulate?

