University of Washington Math Hour Olympiad, 2018

Grades 8-10

6. The Martian alphabet has M letters. Marvin writes down a word and notices that within every sub-word (a contiguous stretch of letters) at least one letter occurs an odd number of times. What is the length of the longest possible word he could have written?

7. For a long space journey, two astronauts with compatible personalities are to be selected from 24 candidates. To find a good fit, each candidate was asked 24 questions that required a simple *yes* or *no* answer.

Two astronauts are compatible if exactly 12 of their answers matched (that is, both answered *yes* or both answered *no*). Miraculously, every pair of these 24 astronauts was compatible!

Show that there were exactly 12 astronauts whose answer to the question "Can you repair a flux capacitor?" was exactly the same as their answer to the question "Are you afraid of heights?" (that is, *yes* to both or *no* to both).