

University of Washington Math Hour Olympiad, 2015

Grades 6-7

1. A party is attended by ten people (men and women). Among them is Pat, who always lies to people of the opposite gender and tells the truth to people of the same gender.

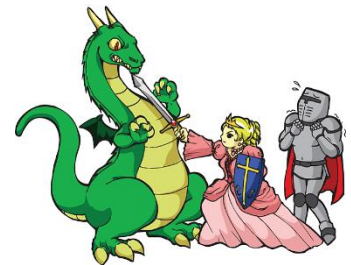
Pat tells five of the guests: "There are more men than women at the party."

Pat tells four of the guests: "There are more women than men at the party."

Is Pat a man or a woman?

2. Once upon a time in a land far, far away there lived 100 knights, 99 princesses, and 101 dragons. Over time, knights beheaded dragons, dragons ate princesses, and princesses poisoned knights. But they always obeyed an ancient law that prohibits killing any creature who has killed an odd number of others.

Eventually only one creature remained alive. Could it have been a knight? A dragon? A princess?



3. The numbers $1 \circ 2 \circ 3 \circ 4 \circ 5 \circ 6 \circ 7 \circ 8 \circ 9 \circ 10$ are written in a line. Alex and Vicky play a game, taking turns inserting either an addition or a multiplication symbol between adjacent numbers. The last player to place a symbol wins if the resulting expression is odd and loses if it is even. Alex moves first. Who wins?

(Remember that multiplication is performed before addition.)

$$1 \times 2 + 3 + 4 + 5 \times 6 \times 7 \times 8 + 9 + 10 = ?$$

4. A chess tournament had 8 participants. Each participant played each other participant once. The winner of a game got 1 point, the loser 0 points, and in the case of a draw each got $\frac{1}{2}$ a point. Each participant scored a different number of points, and the person who got 2nd prize scored the same number of points as the 5th, 6th, 7th and 8th place participants combined.

Can you determine the result of the game between the 3rd place player and the 5th place player?



5. One hundred gnomes sit in a circle. Each gnome gets a card with a number written on one side and a different number written on the other side. Prove that it is possible for all the gnomes to lay down their cards so that no two neighbors have the same numbers facing up.