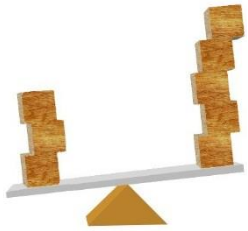
University of Washington Math Hour Open Olympiad, 2013
Grades 8-10

1. Pirate Jim had 8 boxes with gun powder weighing 1, 2, 3, 4, 5, 6, 7, and 8 pounds (the weight is printed on top of every box). Pirate Bob hid a 1-pound gold bar in one of these boxes. Pirate Jim has a balance scale that he can use, but he cannot open any of the boxes. Help him find the box with the gold bar using two weighings on the balance scale.



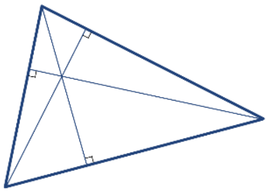
2. James Bond will spend one day at Dr. Evil's mansion to try to determine the answers to two questions:

a) *Is Dr. Evil at home?*   b) *Does Dr. Evil have an army of ninjas?*

The parlor in Dr. Evil's mansion has three windows. At noon, Mr. Bond will sneak into the parlor and use open or closed windows to signal his answers. When he enters the parlor, some windows may already be opened, and Mr. Bond will only have time to open or close one window (or leave them all as they are).

Help Mr. Bond and Moneypenny design a code that will tell Moneypenny the answers to both questions when she drives by later that night and looks at the windows. Note that Moneypenny will not have any way to know which window Mr. Bond opened or closed.

3. Suppose that you have a triangle in which all three side lengths and all three heights are integers. Prove that if these six lengths are all different, there cannot be four prime numbers among them.



4. Fred and George have designed the Amazing Maze, a 5×5 grid of rooms, with Adorable Doors in each wall between rooms. If you pass through a door in one direction, you gain a gold coin. If you pass through the same door in the opposite direction, you lose a gold coin. The brothers designed the maze so that if you ever come back to the room in which you started, you will find that your money has not changed.

Ron entered the northwest corner of the maze with no money. After walking through the maze for a while, he had 8 shiny gold coins in his pocket, at which point he magically teleported himself out of the maze. Knowing this, can you determine whether you will gain or lose a coin when you leave the central room through the north door?

5. Bill and Charlie are playing a game on an infinite strip of graph paper. On Bill's turn, he marks two empty squares of his choice (not necessarily adjacent) with crosses. Charlie, on his turn, can erase any number of crosses, as long as they are all adjacent to each other. Bill wants to create a line of 2013 crosses in a row. Can Charlie stop him?