

Slicing Space

Finding patterns can make hard problems easier!

In arithmetic:

$$1 + 2 + 3 + 4 + \dots + 100 = x = ?$$

$$100 + 99 + 98 + 97 + \dots + 1 = x$$

$$\underbrace{101 + 101 + 101 + 101 + \dots + 101}_{100 \text{ times}} = 2x$$

100 times

$$x = \frac{100 \cdot 101}{2} = \boxed{5050}$$

↑

$$\Rightarrow 2x = 100 \cdot 101$$

Same idea: $1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$

In geometry?

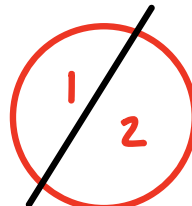
Q: How many pieces of pizza can be made from

# cuts	# pieces
0	1
1	2
2	4
3	7 (?)

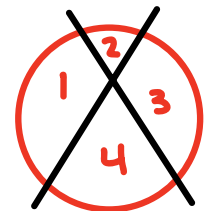
n (straight) cuts?



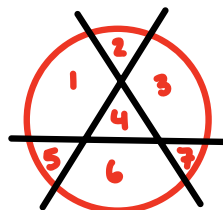
0 cuts



1 cut



2 cuts



3 cuts \rightarrow 7 pieces
Can we do better?

An easier question for warm up:

Q: How many pieces of breadstick can be made from n cuts?

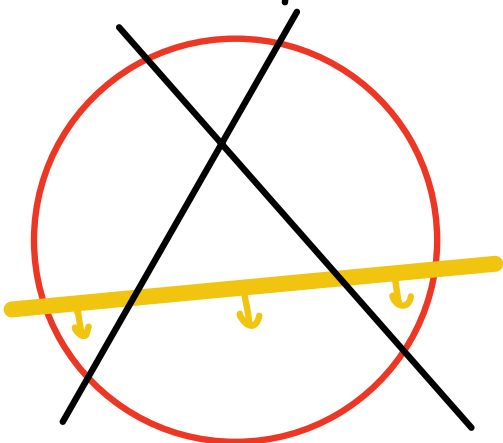
# cuts	# pieces
0	1
1	2
2	3
3	4
\vdots	\vdots
n	$n+1$



Every cut adds one piece!

n cuts \leadsto $n+1$ pieces

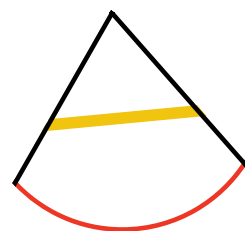
Back to pizza: imagine making one of the pizza



Cuts with a breadstick

Old cuts cut up the breadstick

Each breadstick piece creates new piece of pizza!

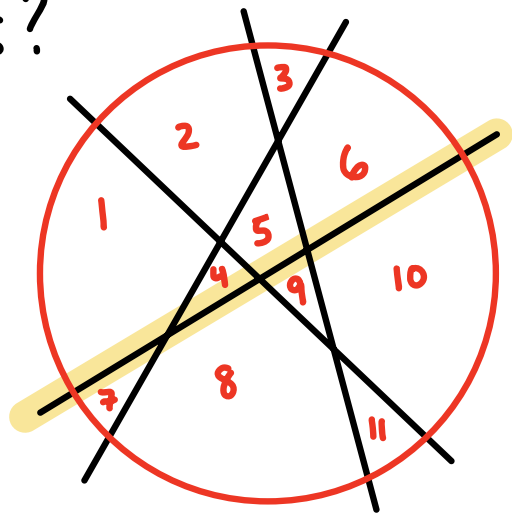


$$\begin{aligned} \# \text{ pizza pieces with 3 cuts} &= \# \text{ pizza pieces with 2 cuts} + \# \text{ breadstick pieces with 2 cuts} \\ &= 4 + 3 \end{aligned}$$

More generally:

$$\# \text{ pizza pieces with } n \text{ cuts} = \# \text{ pizza pieces with } n-1 \text{ cuts} + \# \text{ breadstick pieces with } n-1 \text{ cuts}$$

4 cuts?



pizza pieces with 3 cuts + # stick pieces with 3 cuts
 $7 + 4 = 11$

General formula?

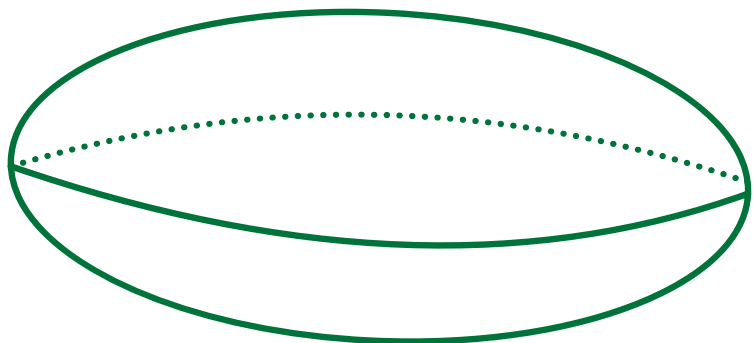
# cuts	# pieces
0	1
1	2
2	4
3	7
4	11
⋮	⋮
n	$1 + 1 + 2 + 3 + 4 + \dots + n = 1 + \frac{n(n+1)}{2}$

$+1$
 $+2$
 $+3$
 $+4$

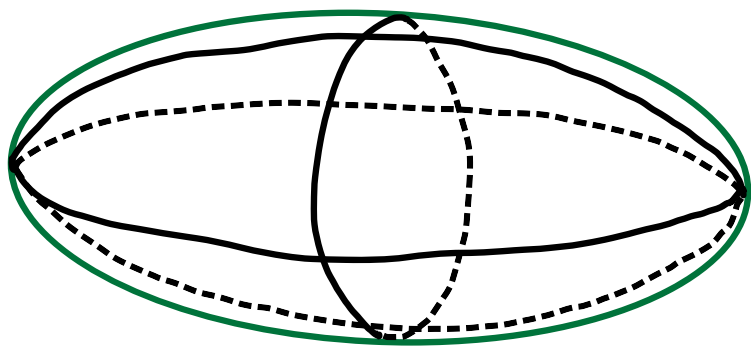
$+1+2+3+4$
 $= \frac{4(4+1)}{2} = 10$

Harder question:

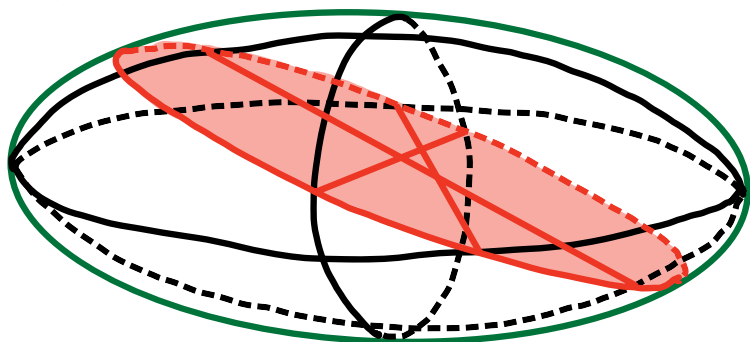
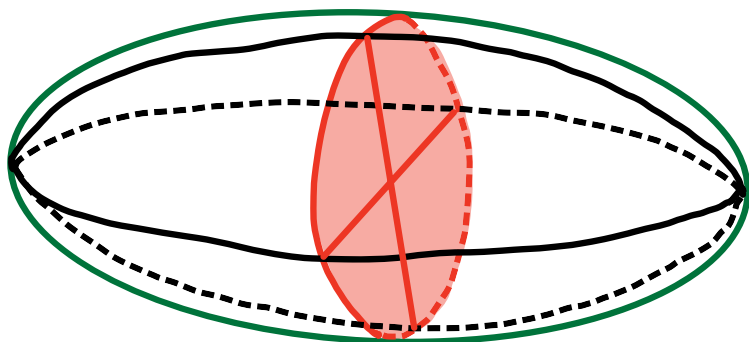
Q: How many pieces of watermelon can be made from n (straight) cuts?



# cuts	# pieces
0	1
1	2
2	4
3	8
4	?



Imagine making last cut with a pizza!



# cuts	# pieces
0	1
1	2
2	4
3	8
4	15
5	26
⋮	⋮
n	(?)

watermelon pieces with n cuts
 = # watermelon pieces with $n-1$ cuts
 + # pizza pieces with $n-1$ cuts

More questions:

- How many crust/rind-less pieces are there?
- What about 4-dim'l food?
or d -dim'l food?!?

