

Strategic Game Theory

Games!

Board games!

Sport games!

Video games!

More video games!

Politics and Finance

Today: play a series of games, see if we can figure out how to win!

Game 1: 21

- ▶ Two players take turns counting
- ▶ Start from “1”
- ▶ Each player can count at most **three** numbers
- ▶ First player that says “21” wins

Game 1: 21

- ▶ Two players take turns counting
- ▶ Start from “1”
- ▶ Each player can count at most **three** numbers
- ▶ First player that says “21” wins
 - ▶ Alice: 1, 2, 3
 - ▶ Bob: 4, 5
 - ▶ Alice: 6, 7
 - ▶ Bob: 8, 9, 10
 - ▶ Alice: 11
 - ▶ Bob: 12, 13, 14
 - ▶ Alice: 15, 16
 - ▶ Bob: 17, 18, 19
 - ▶ Alice: 20, 21
 - ▶ Alice wins

Let's try it!

What's the trick here?

Modification 1

- ▶ Same rules, but whoever first counts to “22” wins

Modification 1

- ▶ Same rules, but whoever first counts to “22” wins
- ▶ What about “23”?

Modification 1

- ▶ Same rules, but whoever first counts to “22” wins
- ▶ What about “23”?
- ▶ What about “24”? Who should go first?

Modification 1

- ▶ Same rules, but whoever first counts to “22” wins
- ▶ What about “23”?
- ▶ What about “24”? Who should go first?
- ▶ What about “2023”?

Modification 2

- ▶ Same rules, but now each player can count up to **four** numbers each time

Modification 2

- ▶ Same rules, but now each player can count up to **four** numbers each time
- ▶ What about **five** numbers each time?

Final test for Game 1

- ▶ Each player can count up to **nine** numbers, first one to “2023” wins

Game 2: Grid game

- ▶ 4×5 board
- ▶ Start by placing a rock at the bottom left
- ▶ Each turn, a player can move the rock directly above, directly to the right, or diagonally up and above
- ▶ Whoever moves the rock at the top right corner wins

Game 2 modification

- ▶ What about a 5×5 board?

Game 2 modification

- ▶ What about a 5×5 board?
- ▶ What about a 8×8 board?

Game 2 modification

- ▶ What about a 5×5 board?
- ▶ What about a 8×8 board?
- ▶ What about a 100×100 board?

Game 3: Piles of stones

- ▶ Two piles of stones: 5, 7
- ▶ Each turn, a player can pick a pile, and take any number of stones from it
- ▶ Player who takes the last stone wins

Game 3 modification

- ▶ Three piles of stones: 3, 5, 7
- ▶ Player who takes the last stone wins