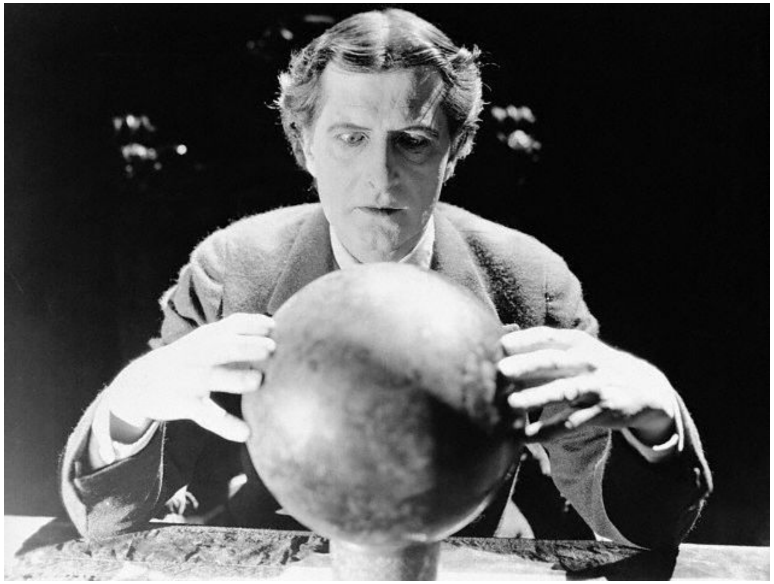
The Mathematics of Weather Forecasting

Cliff Mass
Atmospheric Sciences



Weather forecasting is as old as our species

And It Began As a Non-Mathematical Activity

Weather Proverbs and Sayings

- "Red sky at night, sailor's delight. Red sky in the morning, sailor take warning."
- "Clear moon, frost soon."
- "Halo around the sun or moon, rain or snow soon."

Aristotle's Student, Theophrastus:

"If the breezes come from the east or south, rain is indicated; if from the west or north, cold weather." (generally true)

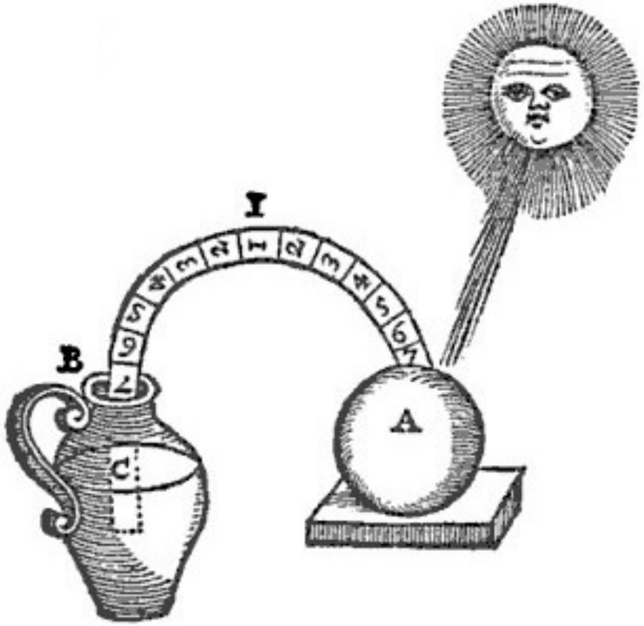
Ancient Weather Instruments

- Wind vanes were perhaps the most ancient meteorological instruments.
- Mesopotamian and Sumerian documents, dating back nearly 4,000 years, describe primitive wind vanes, and streamers.

Tower of the Winds, 50 BCE

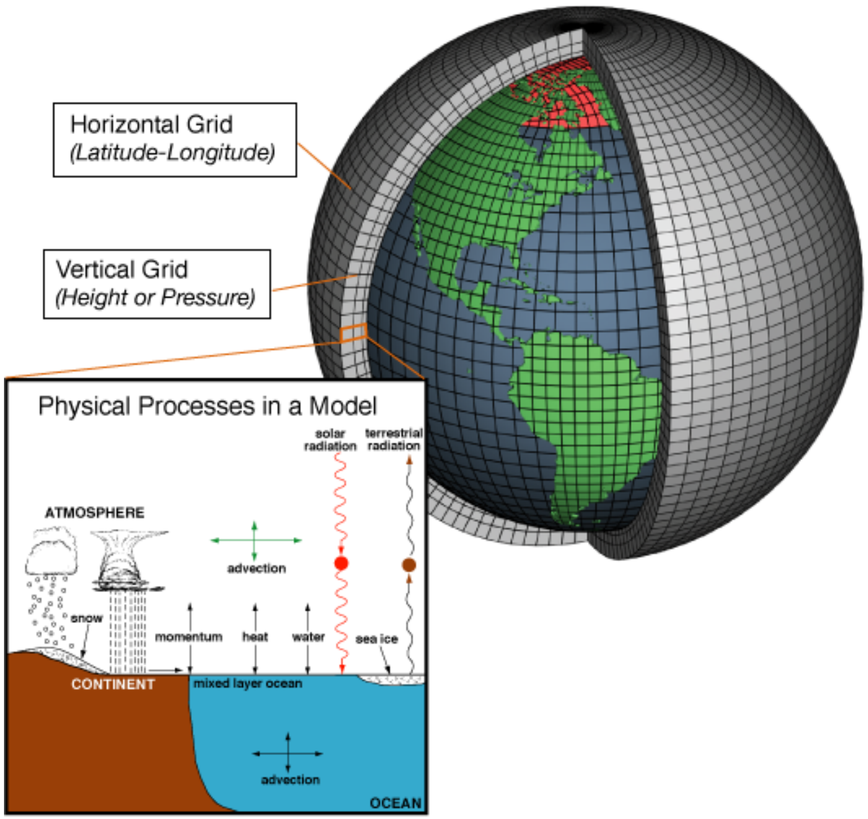
Ancient Weather Instruments

- Bowls were used as rain gauges (India, 4th century BC)
- First thermometer (Philo of Byzantium, 240 BC)

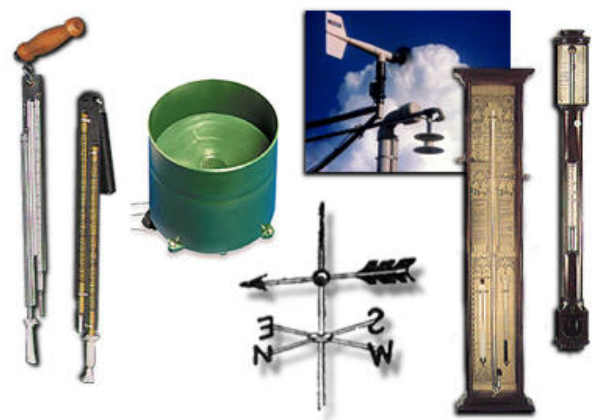


Philo's Thermometer

The Transition to Qualitative Modern Weather Forecasting



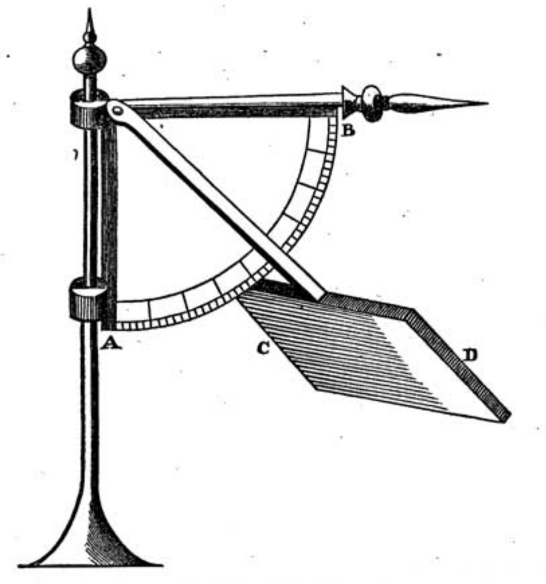
Step 1: You need weather instruments with sufficient accuracy and reproducibility.



1450-1750
The Rise of Weather Instruments

- Around 1600 A.D., a number of experimenters, including Galileo Galilei, developed air-water *thermoscopes*
- The first modern thermometer

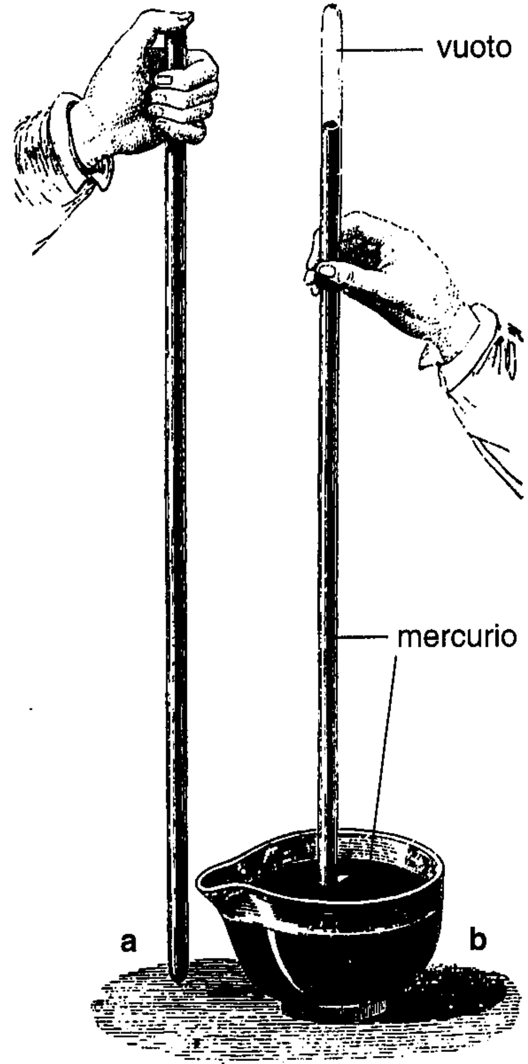
Robert Hooke (1660) Invented the Anemometer to Measure Wind Speed



Hooke, 1660

Anderson, 1840

Torricelli Devised the Mercury Barometer to Measure Pressure (1644)



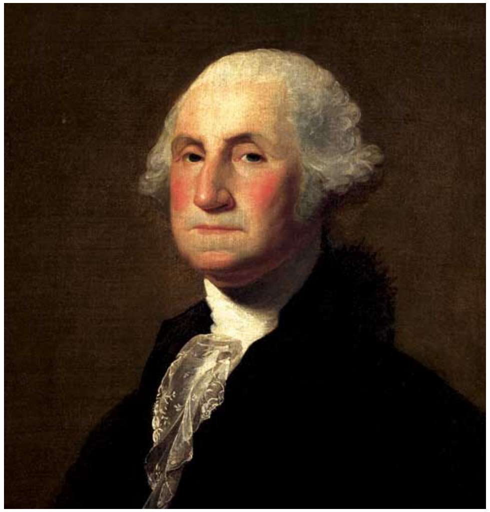
But there was a problem… the absence of universal scales for basic weather parameters such as temperature

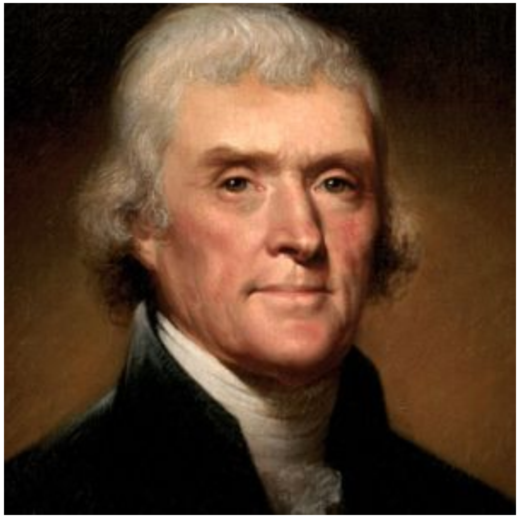
Solved by agreeing to standard units (metric, English units): °C or °F for temperature, knots or meters per second for wind)

By the late 1700's everything was in place: decent instruments and universal scales

- **The weather observing revolution was about to begin**
- **Observing the weather became the high-tech craze of the period**

Early Weather Hobbyists



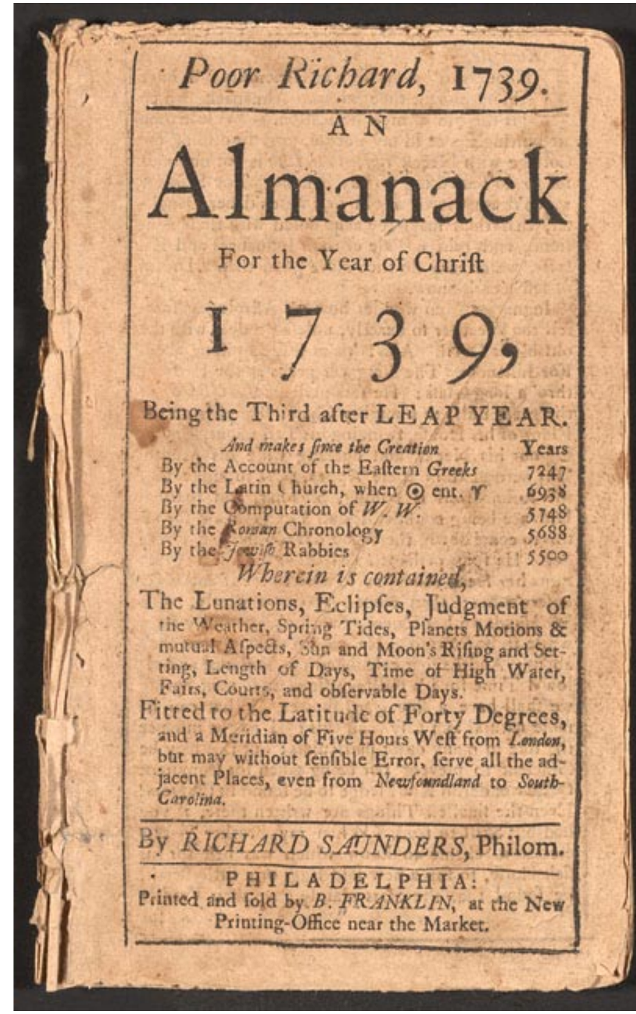


- Thomas Jefferson maintained a nearly unbroken record of weather observations between 1776 and 1816. Four weather observations on July 4, 1776!
- George Washington collected daily weather observations until only a few days before he died.

Franklin Foresaw Modern Weather Prediction

Ben Franklin, the Weather Forecaster

- Franklin concluded, based on the visibility of a 1743 lunar eclipse at various locations, that a storm moved to the northeast.
- Thus, it might be possible to *predict* storm movement if information could move faster than the storms.
- Produced some of the first written weather forecasts in Poor Richard's Almanac


Poor Richard, 1739.
AN
Almanack
For the Year of Chriſt
1739,
Being the Third after LEAP YEAR.

And makes ſince the Creation	Years
By the Account of the Eaſtern *Greeks*	7247
By the Latin Church, when ☉ ent. ♈	6938
By the Computation of *W. W.*	5748
By the *Roman* Chronology	5688
By the *Jewiſh* Rabbies	5500

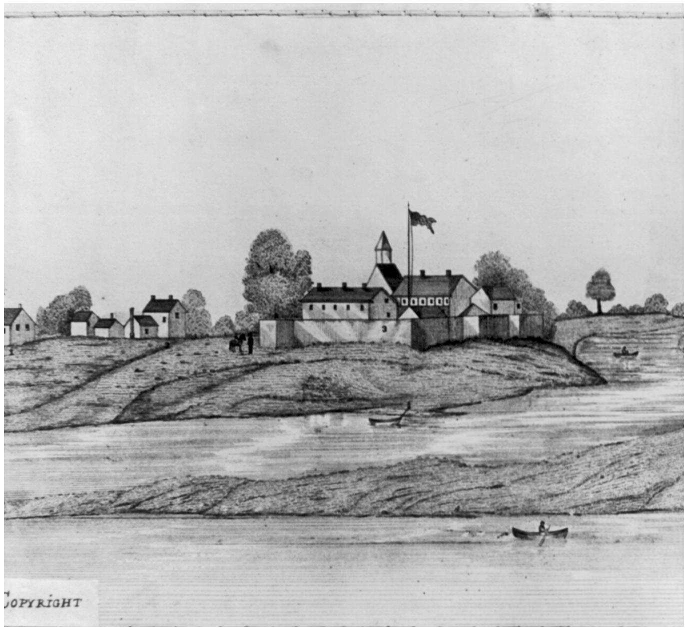
Wherein is contained,
The Lunations, Eclipſes, Judgment of the Weather, Spring Tides, Planets Motions & mutual Aſpects, Sun and Moon's Riſing and Setting, Length of Days, Time of High Water, Fairs, Courts, and obſervable Days.
Fitted to the Latitude of Forty Degrees, and a Meridian of Five Hours Weſt from *London*, but may without ſenſible Error, ſerve all the adjacent Places, even from *Newfoundland* to *South-Carolina*.

By *RICHARD SAUNDERS*, Philom.

PHILADELPHIA:
Printed and ſold by *B. FRANKLIN*, at the New Printing-Office near the Market.

Weather networks were established

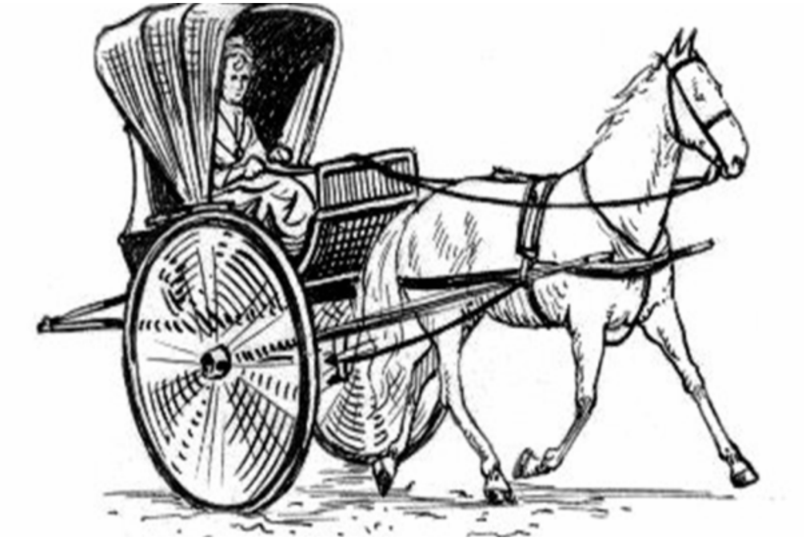
- In 1816 all U.S. army surgeons were required to take three observations daily.
- Other American weather networks were organized under the auspices of the U.S. Navy and the Smithsonian Institution



The Advent of Operational Weather Forecasting

There were lots of weather observations but their value was limited

Without a means for rapid communication, weather maps were created weeks or months after the observations were taken.

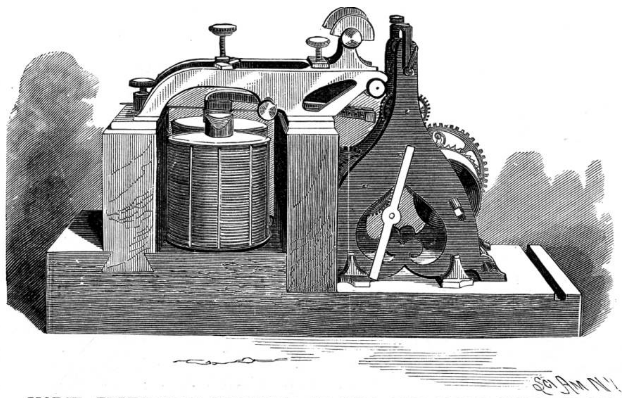


Too slow

The Telegraph Changes Everything

The first practical electrical telegraph was built by Samuel Morse and Alfred Vail in 1837

The Internet of the 1840s



MORSE TELEGRAPH RECEIVER OF 1844—THE FIRST INSTRUMENT RECORDING THE MORSE CODE.

It was immediately recognized that the telegraph made weather prediction possible

Joseph Henry in 1847 provided the vision:

"… extended lines of telegraph will furnish a ready means of warning the more northern and eastern observers to be on the watch for the first appearance of an advancing storm."

The Weather-Telegraph Revolution

- In 1849, the Smithsonian created a network of roughly **two-dozen telegraph operators/observers,** distributing daily weather reports from around the nation.
- **A weather map**, updated daily and based on telegraphic reports, was posted in the lobby of the Smithsonian Institution and Washington Evening Star newspaper published it in each issue.

Operational Weather Prediction Begins

- With telegraphic weather information, **governments around the globe began to establish operational meteorological services**.
- In 1854 Admiral Fitzroy, famed captain of Darwin's ship the Beagle, was appointed head of the new British Meteorological Service
- In 1861 began issuing daily forecasts and storm warnings.

Admiral Robert Fitzroy
Created "*weather forecast*"

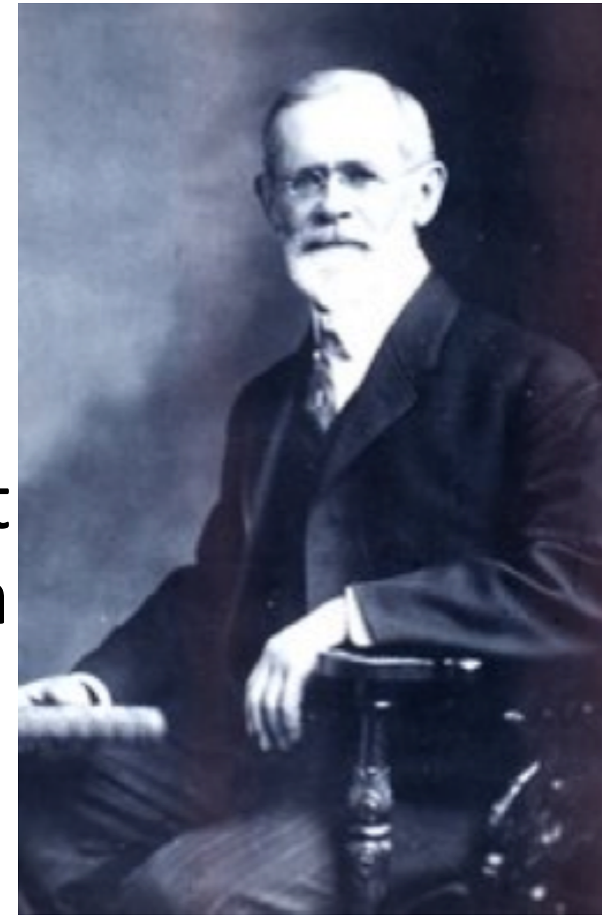
Some Doubters

French physicist François Arago, the Director of the Paris Observatory

*"Whatever may be the progress of sciences, **never** will observers that are trustworthy, and careful of their reputation, venture to foretell the state of the weather."*

U.S. Moves Ahead in Weather Forecasting

- Meteorologist Cleveland Abbe, released the first U.S. public weather forecast on September 1, 1869 for Cincinnati, Ohio
- In 1870, President Ulysses S. Grant **authorized the establishment of a new national weather service located within the Signal Service of the U.S. Army**

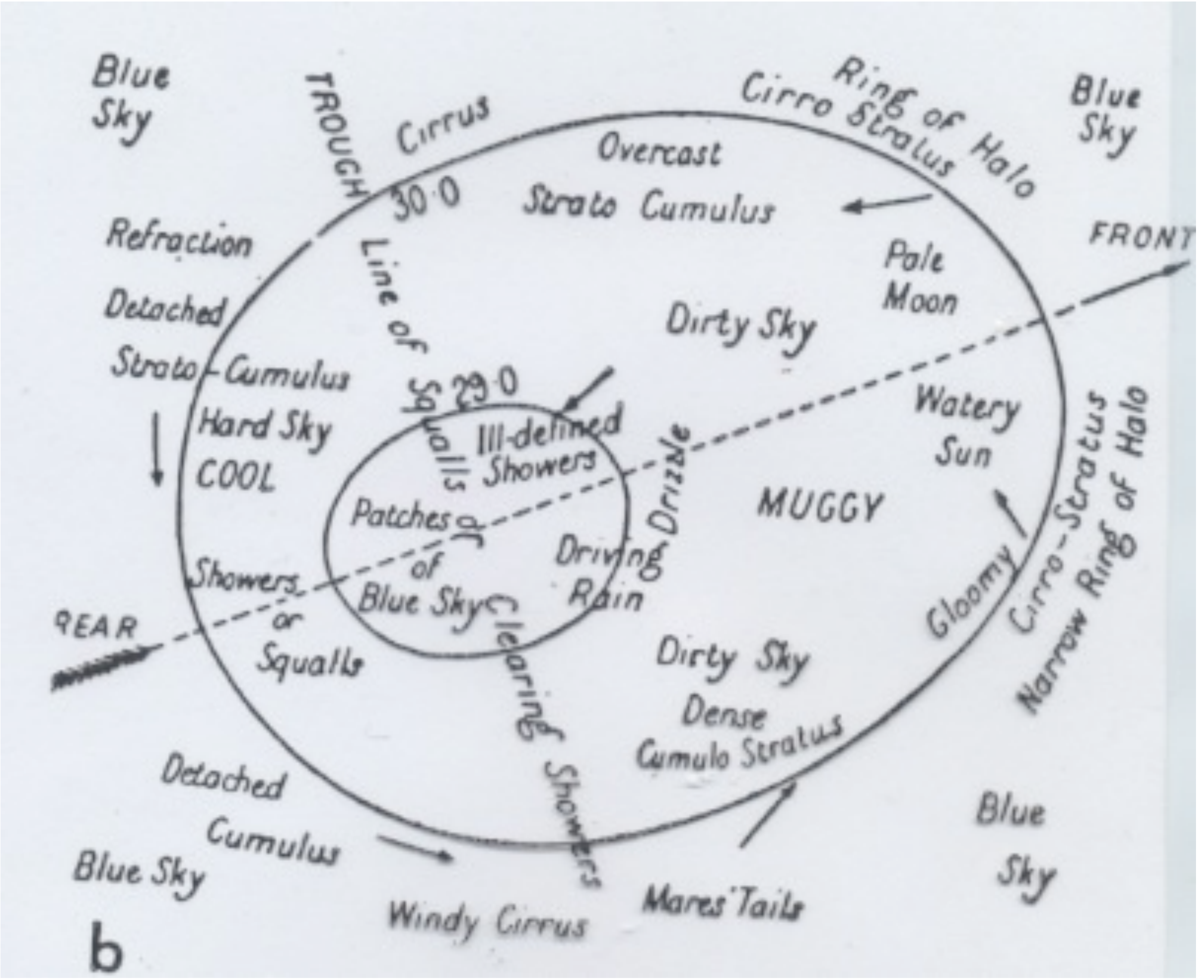


Ol' Probs

(1872) U.S. Operational Weather Maps

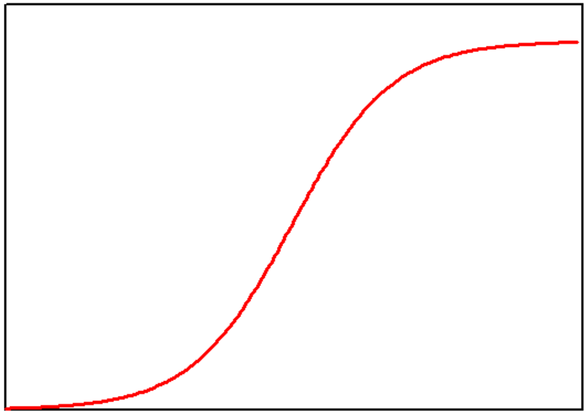
1880’s Forecasting Technology

1890's Atmospheric "Model"

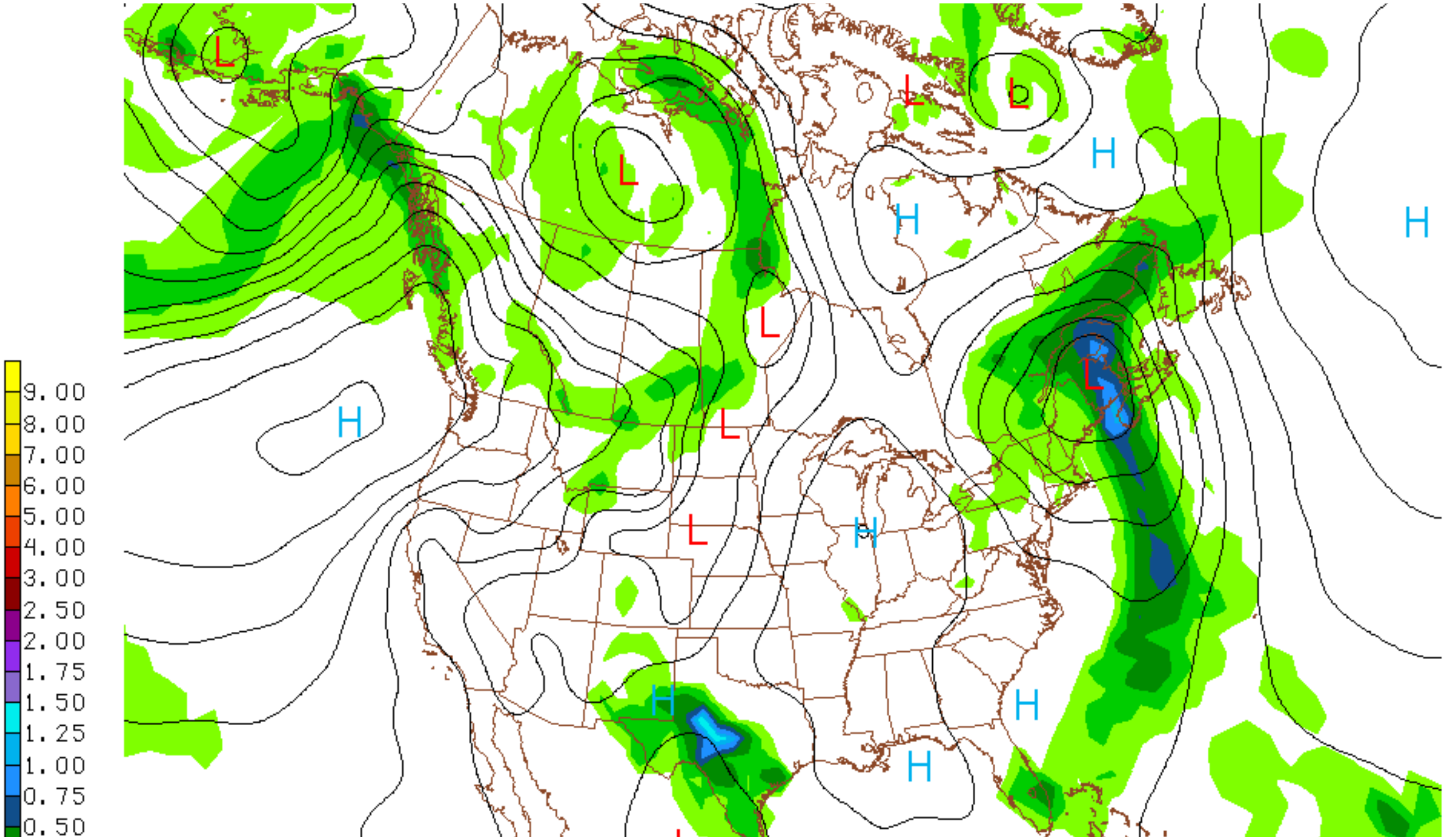


Forecast Skill Levels Off

- Forecasting skill increased during the 1870s and 1880s as more observations became available.
- Little understanding of the evolution of weather systems.
- By the turn of the century, predictive skill had leveled off.
- We needed to get beyond qualitative forecasting.



The Development of Numerical Weather Prediction (NWP)



The Equations Are Revealed

During the 19th century, all the basic equations describing the physics of the atmosphere became known.

- Conservation of momentum
- Conservation of energy
- Conservation of mass
- Conservation of water

$$\frac{d\vec{V}_H}{dt} = \frac{\partial \vec{V}_H}{\partial t} + \vec{V}_H \bullet \vec{\nabla}_H \vec{V} = -\frac{1}{\rho}\vec{\nabla}_H p - f\vec{k} \times \vec{V}_H$$

Known as the The "Primitive Equations"

Partial differential equations that describe basic conservation laws

Wind Forecast Equations

1a. $$\frac{\partial u}{\partial t} = -u\frac{\partial u}{\partial x} - v\frac{\partial u}{\partial y} - \omega\frac{\partial u}{\partial p} + fv - g\frac{\partial z}{\partial x} + F_x$$

1b. $$\frac{\partial v}{\partial t} = -u\frac{\partial v}{\partial x} - v\frac{\partial v}{\partial y} - \omega\frac{\partial v}{\partial p} - fu - g\frac{\partial z}{\partial y} + F_y$$

Continuity Equation

2. $$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial \omega}{\partial p} = 0$$

Temperature Forecast Equation

3. $$\frac{\partial T}{\partial t} = -u\frac{\partial T}{\partial x} - v\frac{\partial T}{\partial y} - \omega\left(\frac{\partial T}{\partial p} - \frac{RT}{c_p p}\right) + \frac{H}{c_p}$$

Moisture Forecast Equation

4. $$\frac{\partial q}{\partial t} = -u\frac{\partial q}{\partial x} - v\frac{\partial q}{\partial y} - \omega\frac{\partial q}{\partial p} + E - P$$

Hydrostatic Equation

5. $$\frac{\partial z}{\partial p} = -\frac{RT}{pg}$$

An Idea is Born

In 1904, Norwegian **Vilhelm Bjerknes** suggested that solving the primitive equations could predict the future atmosphere.

Numerical Weather Prediction

One of the equations used to predict the weather is Newton's Second Law:

$$F = ma$$

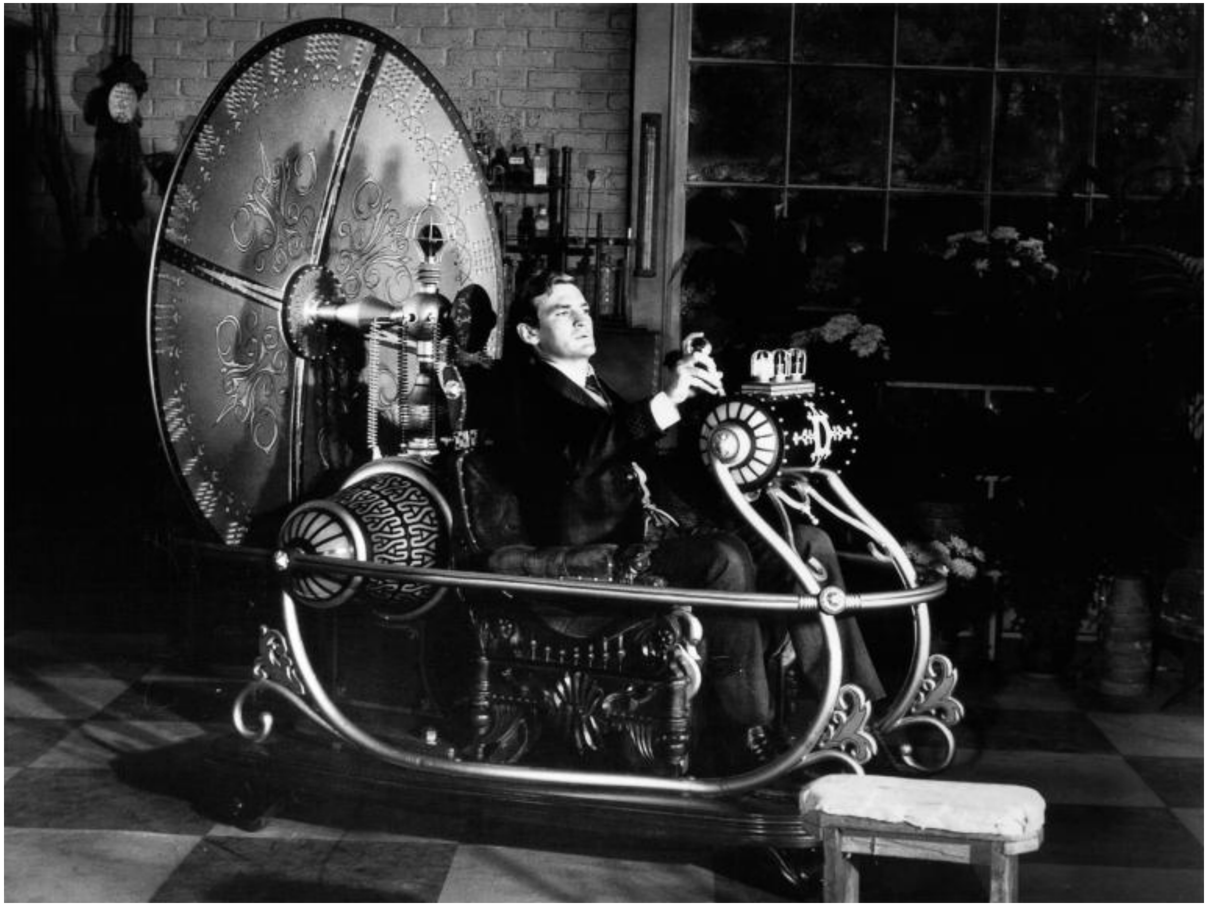
Force = mass x acceleration

Mass is the amount of matter

Acceleration is how velocity changes with time

Force is a push or pull on some object (e.g., gravitational force, pressure forces, friction)

This equation is a time machine!



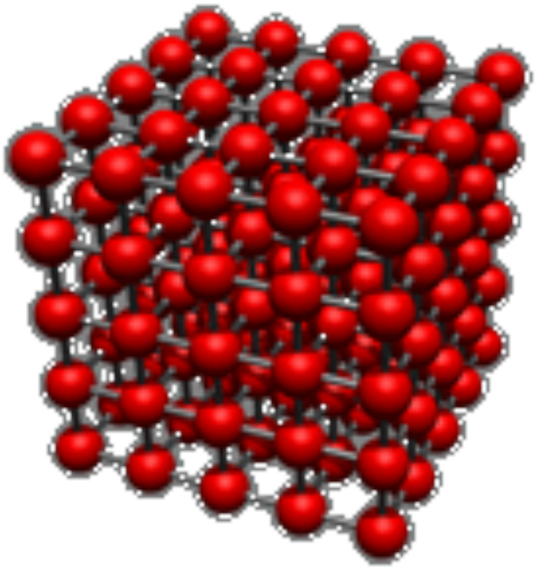
Why Time Machine?

F = ma

- If we can observe the atmosphere, we know **m**, the amount of mass
- With observations can calculate all the forces, **F** (such as forces due to differences in pressure)
- Thus, we can calculate the acceleration, **a**.
- Acceleration is the **change of velocity in time**.
- **THUS WE KNOW THE FUTURE WINDS. Similar equations for temperature and humidity**

Numerical Weather Prediction

- If you can observe the current state of the atmosphere (known as the initialization) , you can predict the future using the equations that describe the physics of the atmosphere.
- These equations can be solved on a three-dimensional grid.

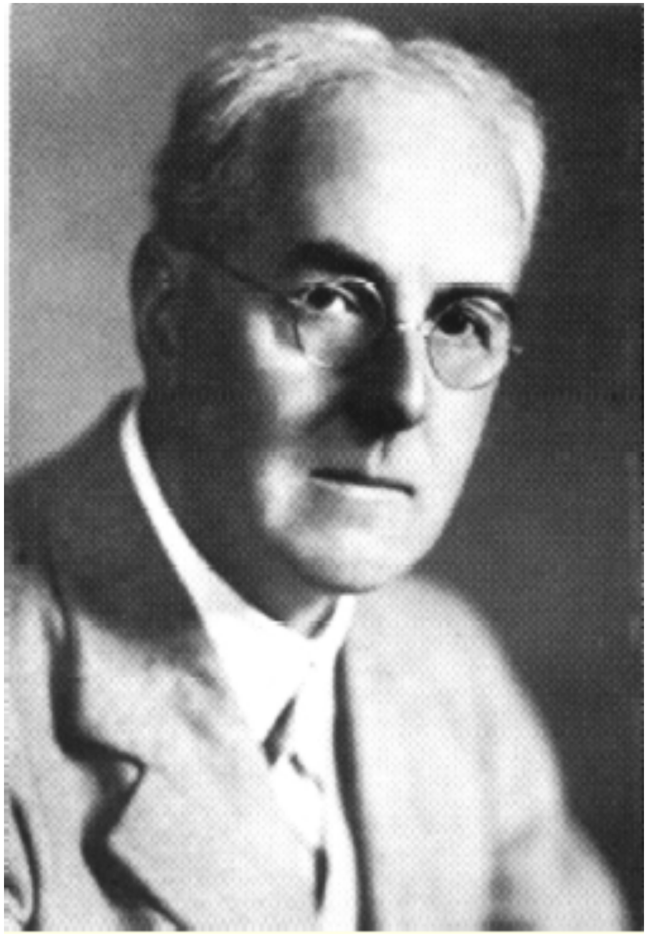


But in the early 20th Century Numerical Weather Prediction Was Thought to be IMPOSSIBLE to accomplish in reality

- No way to do the calculations fast enough (all they had were mechanical calculators)
- Fully three dimensional observations were necessary: all that were available were at the surface!

L. F. Richardson: An Insightful But Unsuccessful Attempt at Numerical Prediction

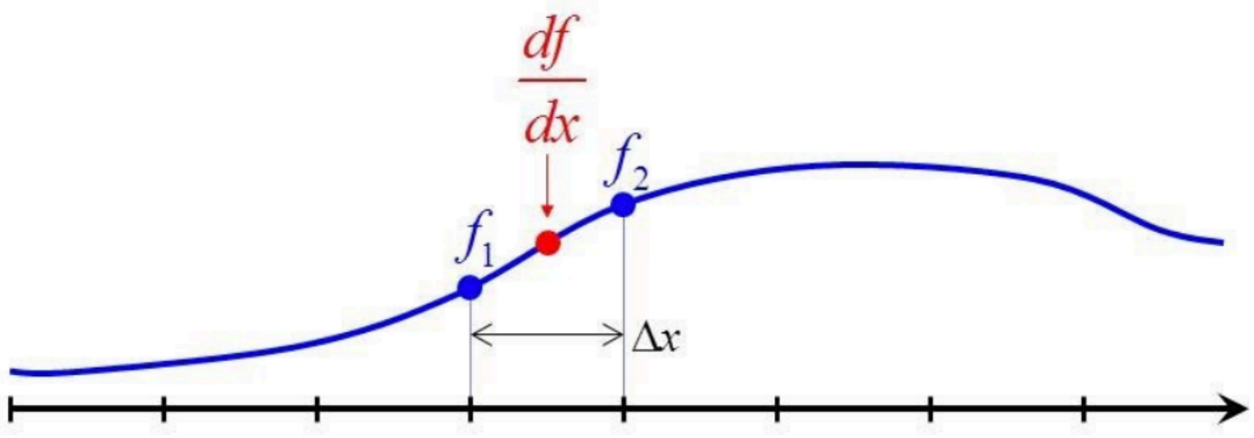
- As a Quaker ambulance driver in WWI he worked on the problem.
- In 1922 Richardson published a book *Weather Prediction by Numerical Process* that described an approach to using the primitive equations: **solving the equations on a grid using finite differences**



Finite differences with variables (like temperature) on a grid of points

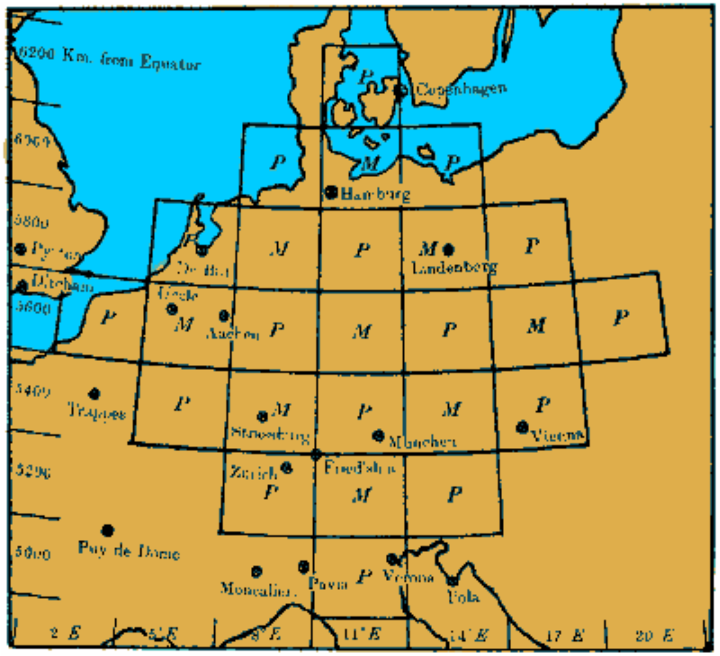
$$\frac{df_{1.5}}{dx} \approx \frac{f_2 - f_1}{\Delta x}$$

second-order accurate first-order derivative



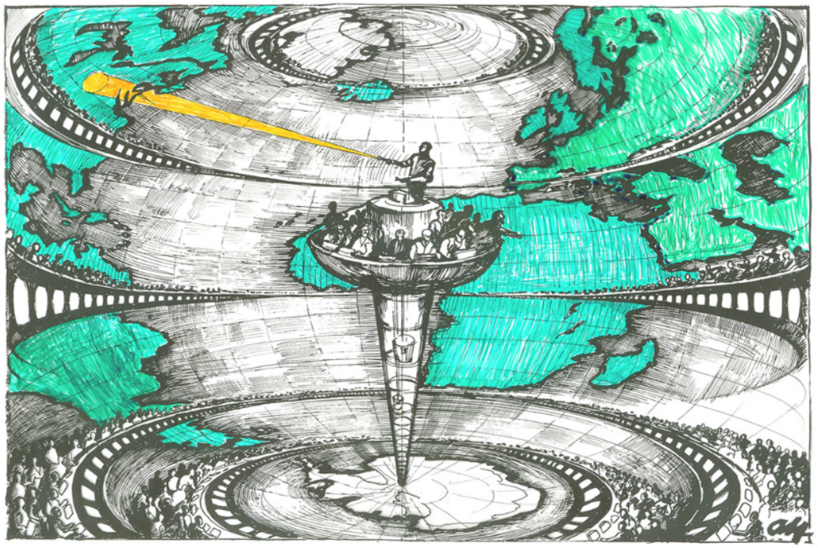
L. F. Richardson

- He attempted to make a numerical forecast using a mechanical calculator
- Unfortunately, the results were not good, probably because of problems with his initial conditions and long time step.



L. F. Richardson

- He imagined a giant theater filled with humans using mechanical calculators…
- Practical NWP had to wait until a way of doing the computations quickly was developed and more data… especially aloft… became available.

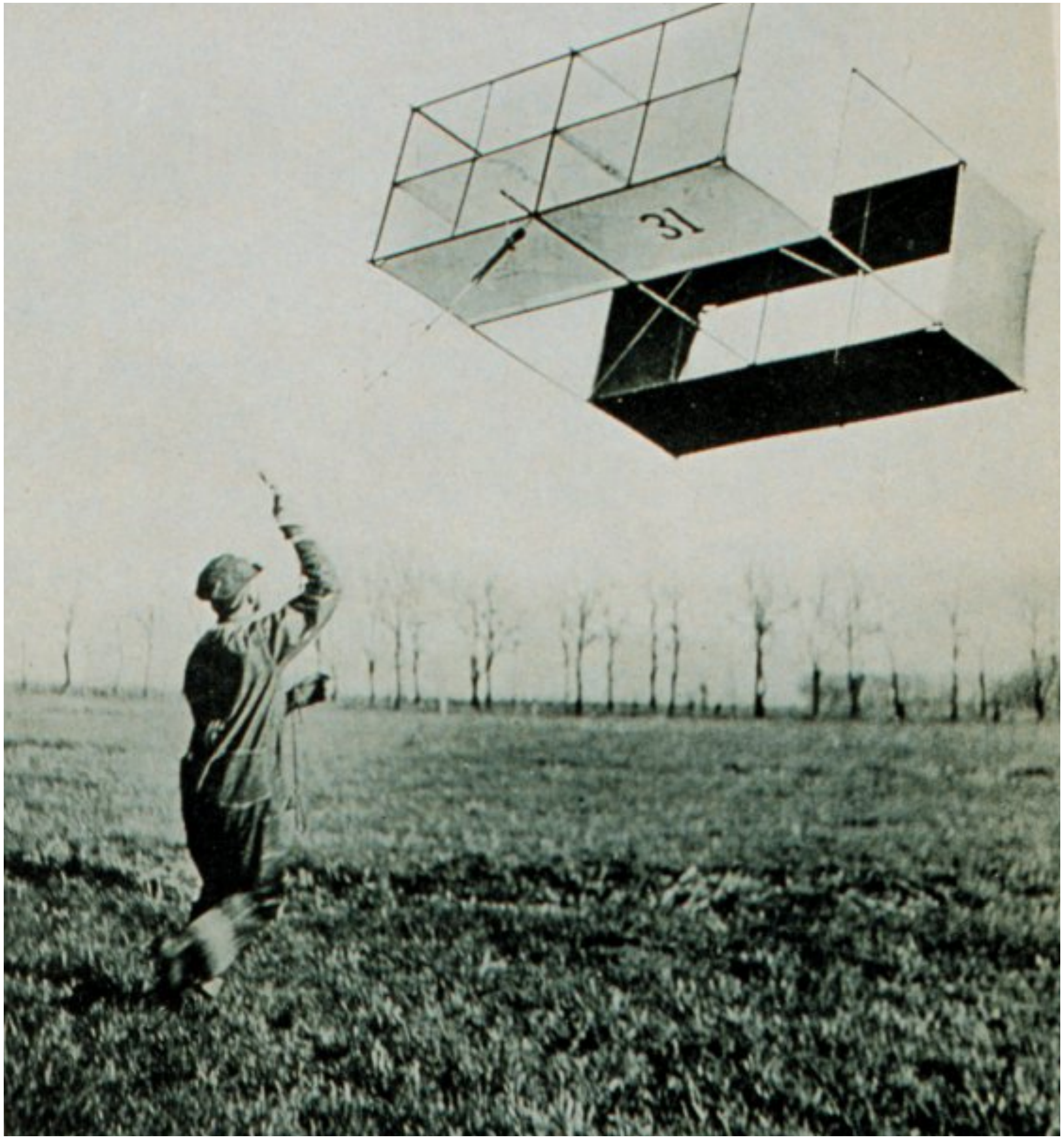


A Lack of Upper Air Observations Was Preventing Numerical Weather Prediction

- The atmosphere is three dimensional and what happens aloft matters.
- In 1920, a very sparse upper-air observational network:
  - Mountain stations
  - Kites and pilot balloons
  - Limited aircraft observations.

Navy bi-plane with meteorgraph on starboard wing strut, taking meteorological measurements for pressure, temperature, and humidity

Weather Kite



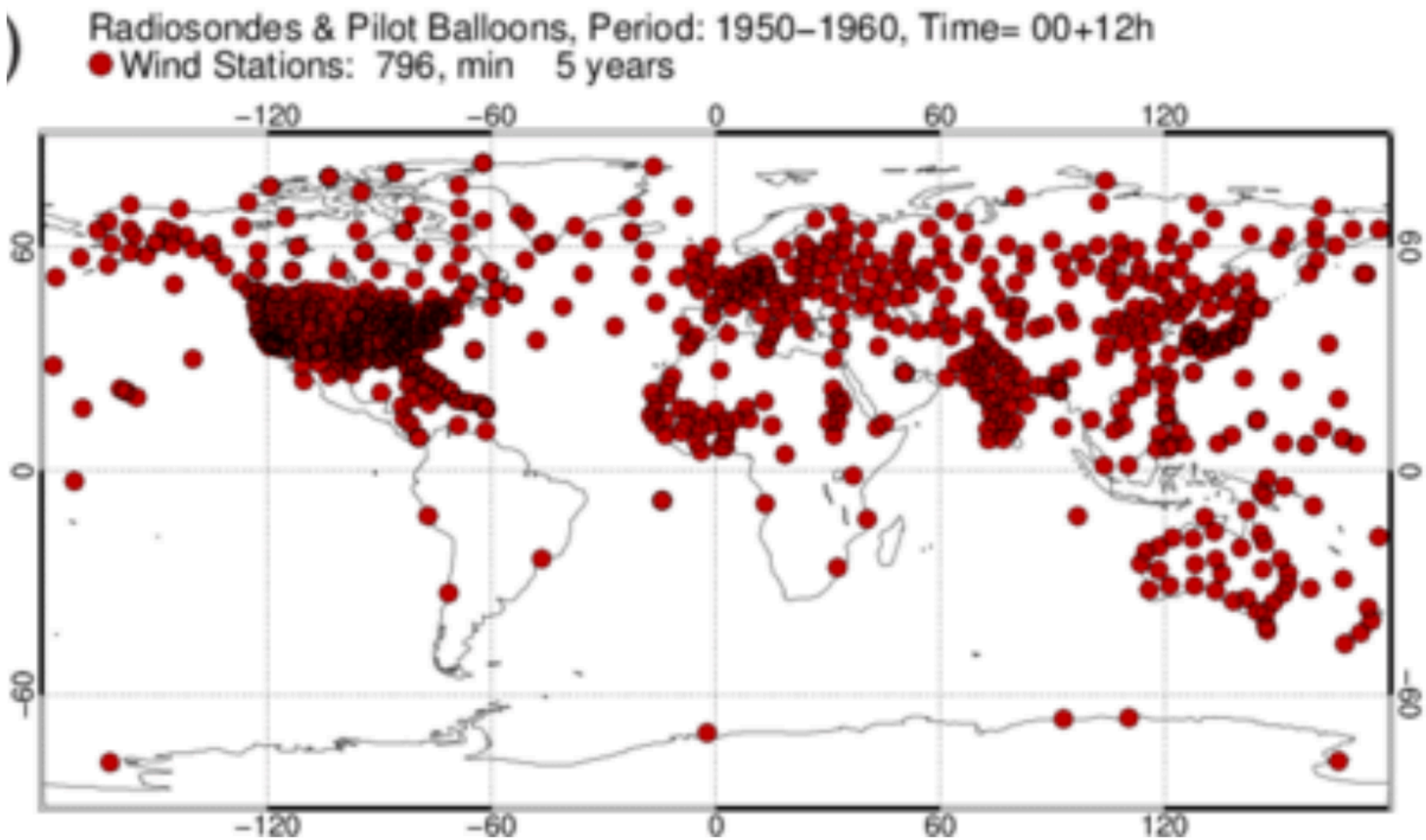
The Big Breakthrough: The Radiosonde

- **A radiosonde is a portable weather station lofted by a balloon.**
- **Sends observations back by radio.**
- **The first instrument launched on January 7, 1929.**

Rapid Expansion of the Upper Air Network During the 1930s and 1940s.

Table 1. Number of soundings for different measurement platforms and time periods.

Platform	pre-1928	1928–37	1938–47	1948–57
Aircraft	1,419	15,363	7,392	0
Kite	29,850	28,501	495	0
Pilot balloon/registering balloon	240,200	1,061,328	3,794,542	5,101,760
Radiosonde	0	1,004	147,099	2,319,339



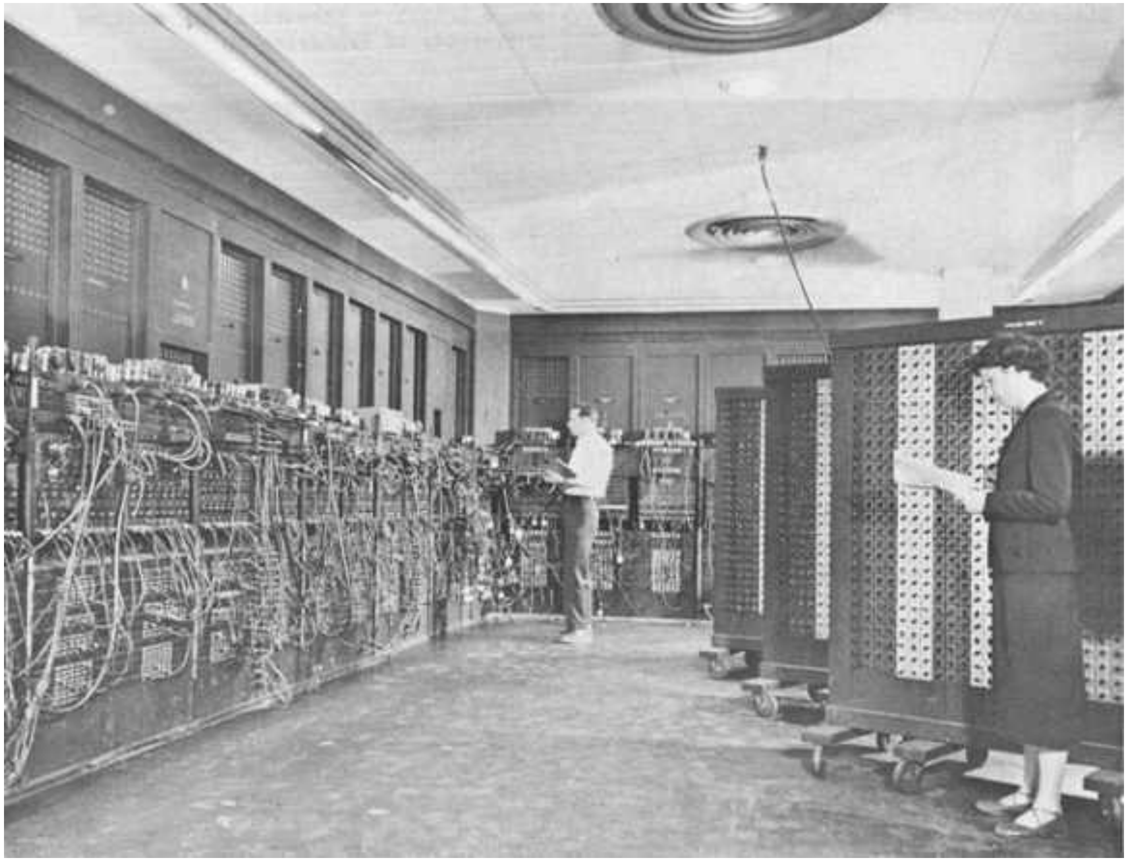
Radiosondes & Pilot Balloons, Period: 1950−1960, Time= 00+12h
Wind Stations: 796, min 5 years
−120
−60
0
60
120
60
0
−60

The Dream of Numerical Weather Prediction Becomes Possible

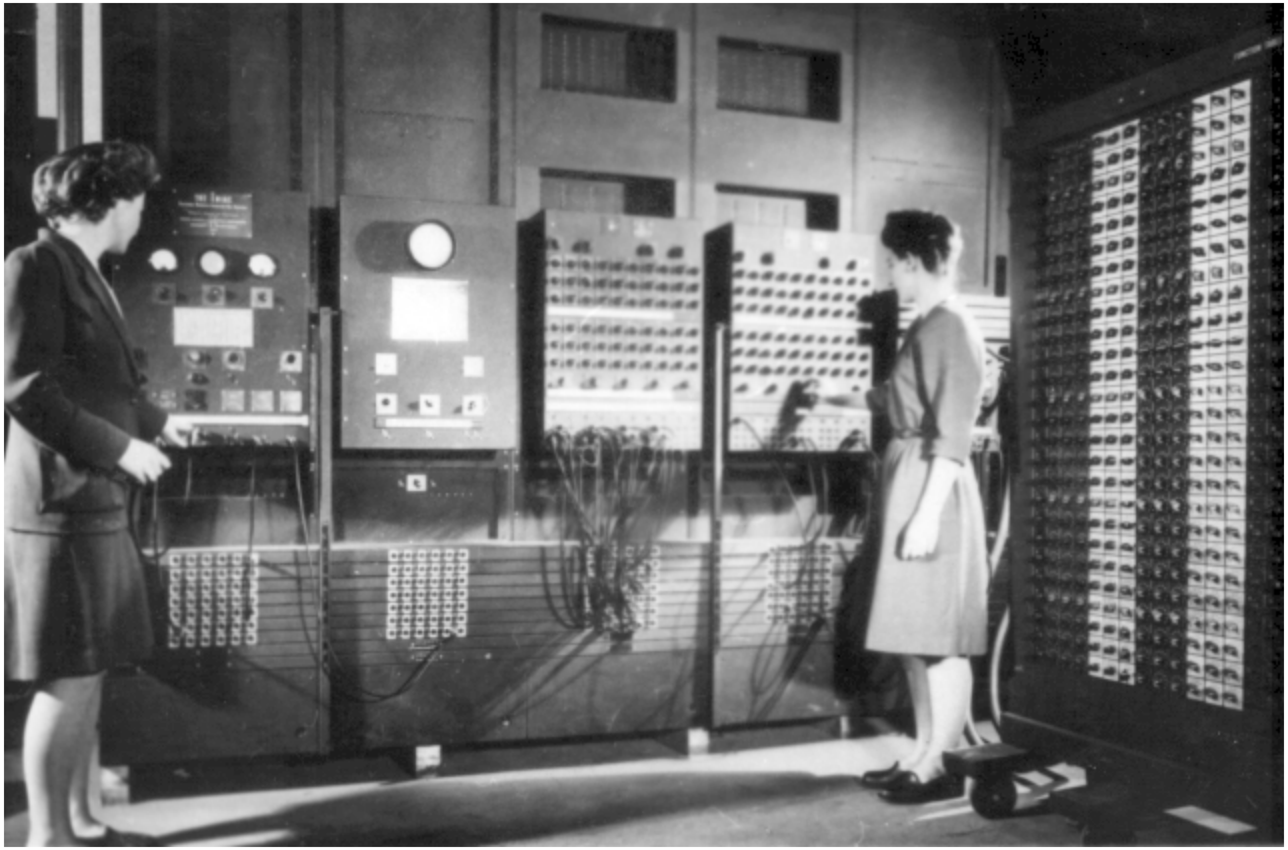
- Facilitated by WWII, by the mid to late 1940's there was an extensive upper air radiosonde network. **Thus, a reasonable 3-D description of the atmosphere was possible.**
- Also during this period digital programmable computers were becoming available …the first being the ENIAC

Programmable Digital Computers

Eniac



Eniac Control Room

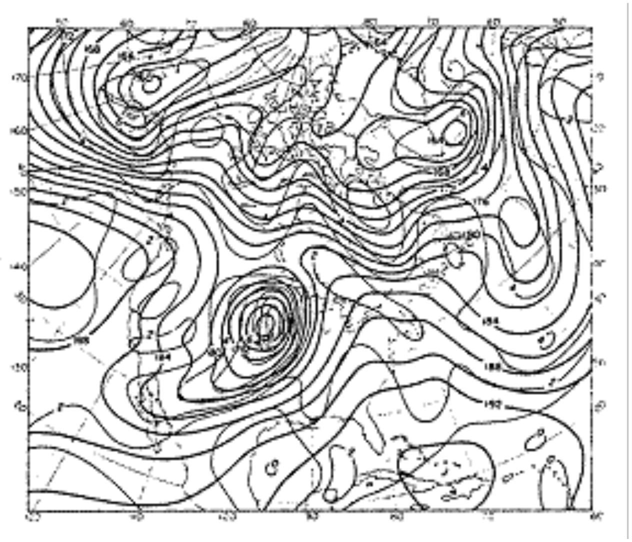


First Numerical Weather Prediction Science Team

FIG. 1. Visitors and participants in the 1950 ENIAC computations (left to right): Harry Wexler, John von Neumann, M. H. Frankel, Jerome Namias, John Freeman, Ragnar Fjörtoft, Francis Reichelderfer, and Jule Charney.

The First Successful Numerical Weather Prediction

- Took place in April 1950, using the ENIAC
- The prediction was for 500-mb height over North America, using a two-dimensional grid with 270 points about 700 km apart.
- One level model The results were clearly superior to human subjective prediction.
- **The NWP era had begun**.



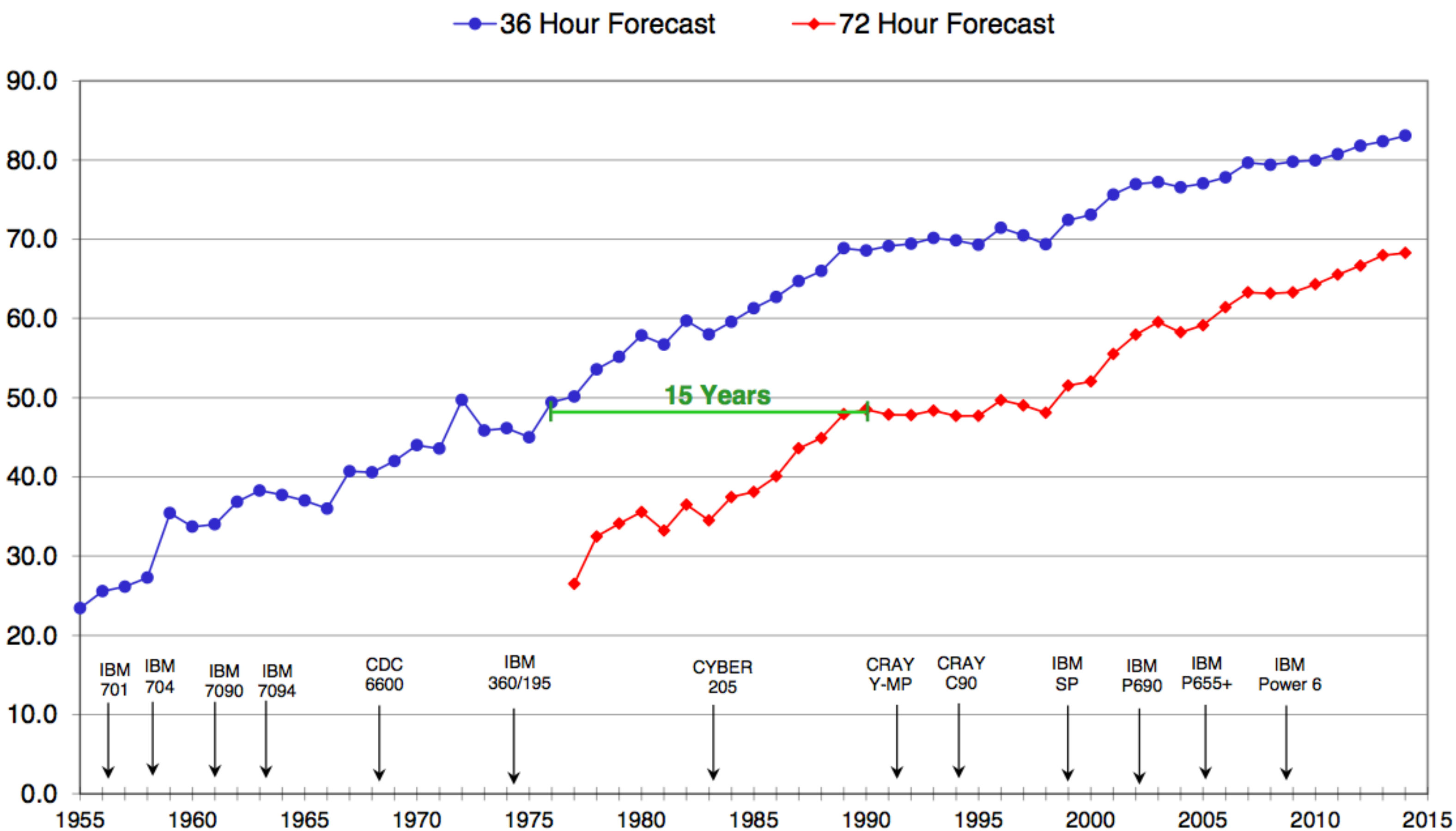
Faster Computers Drove Better Weather Prediction

- **Resolution increases** (distance between grid points decrease): 1958: 380 km, 1985: 80 km, 1995: 40 km, 2000: 22 km, 2002: 12 km, 2008: 4 km
- **Better description of physical processes**. Like clouds and radiation.
- **Better use of observations**: improved **data assimilation**

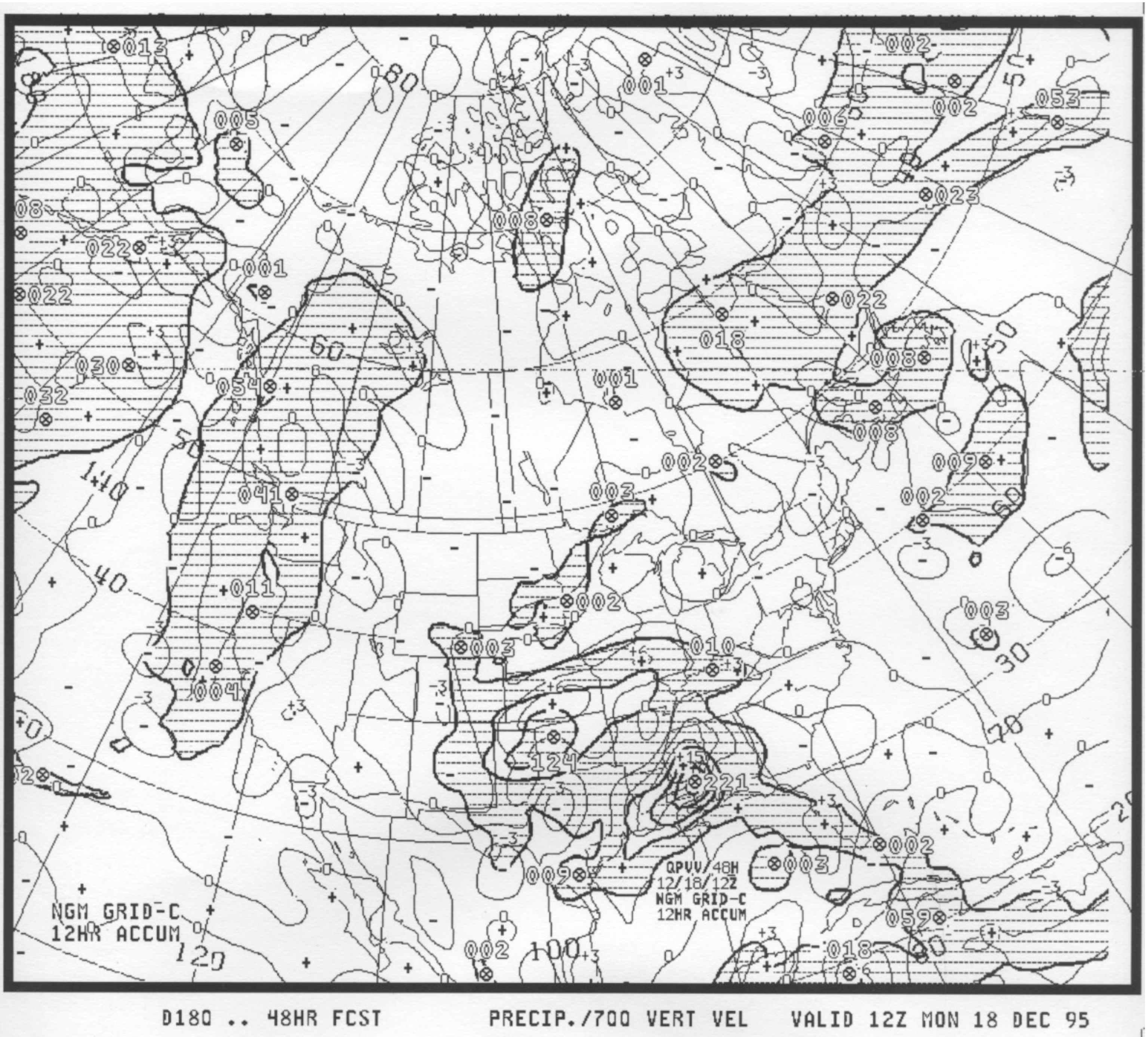
NCEP Operational Forecast Skill

36 and 72 Hour Forecasts @ 500 MB over North America

[100 * (1-S1/70) Method]

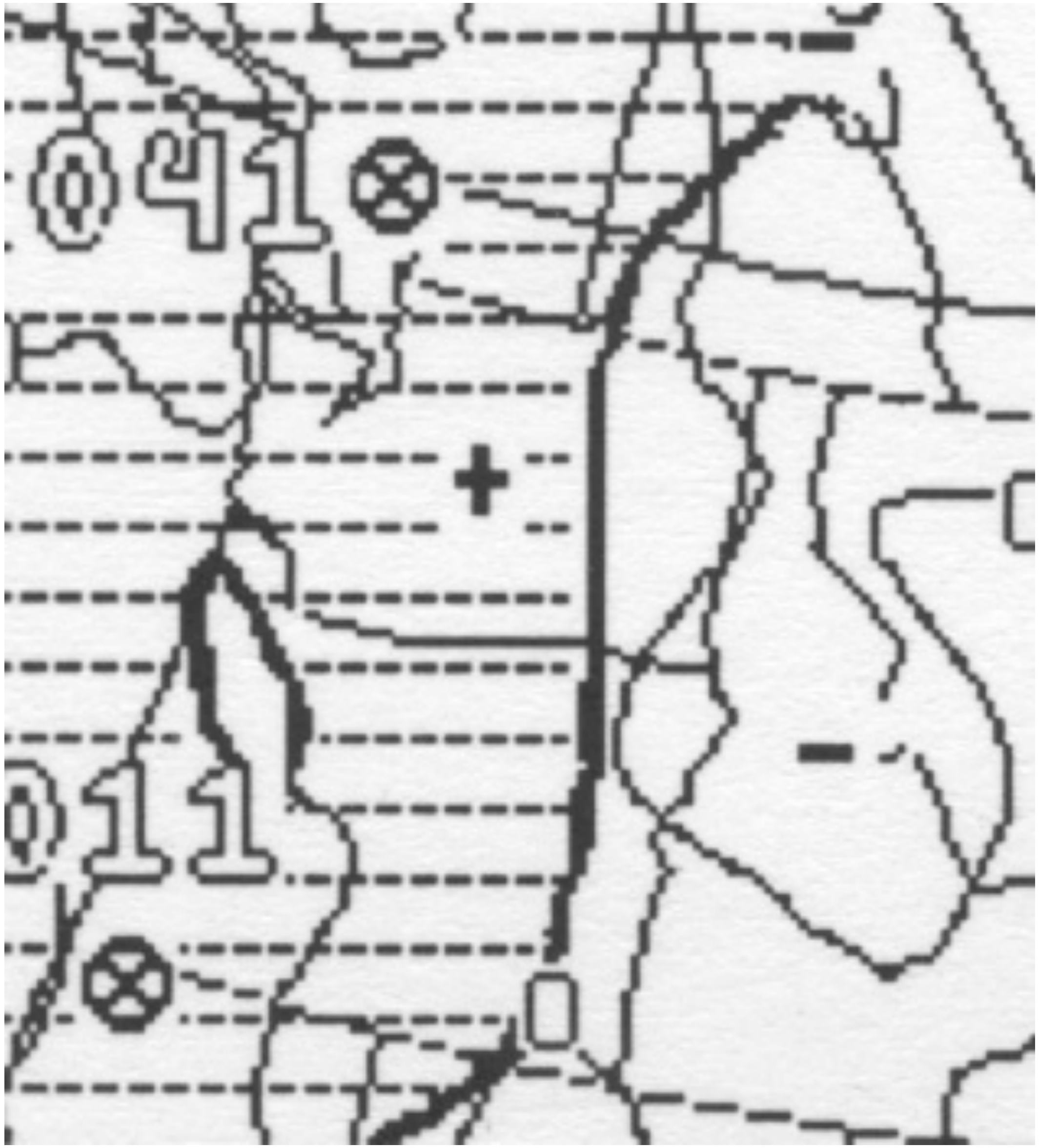


NCEP Central Operations January 2015

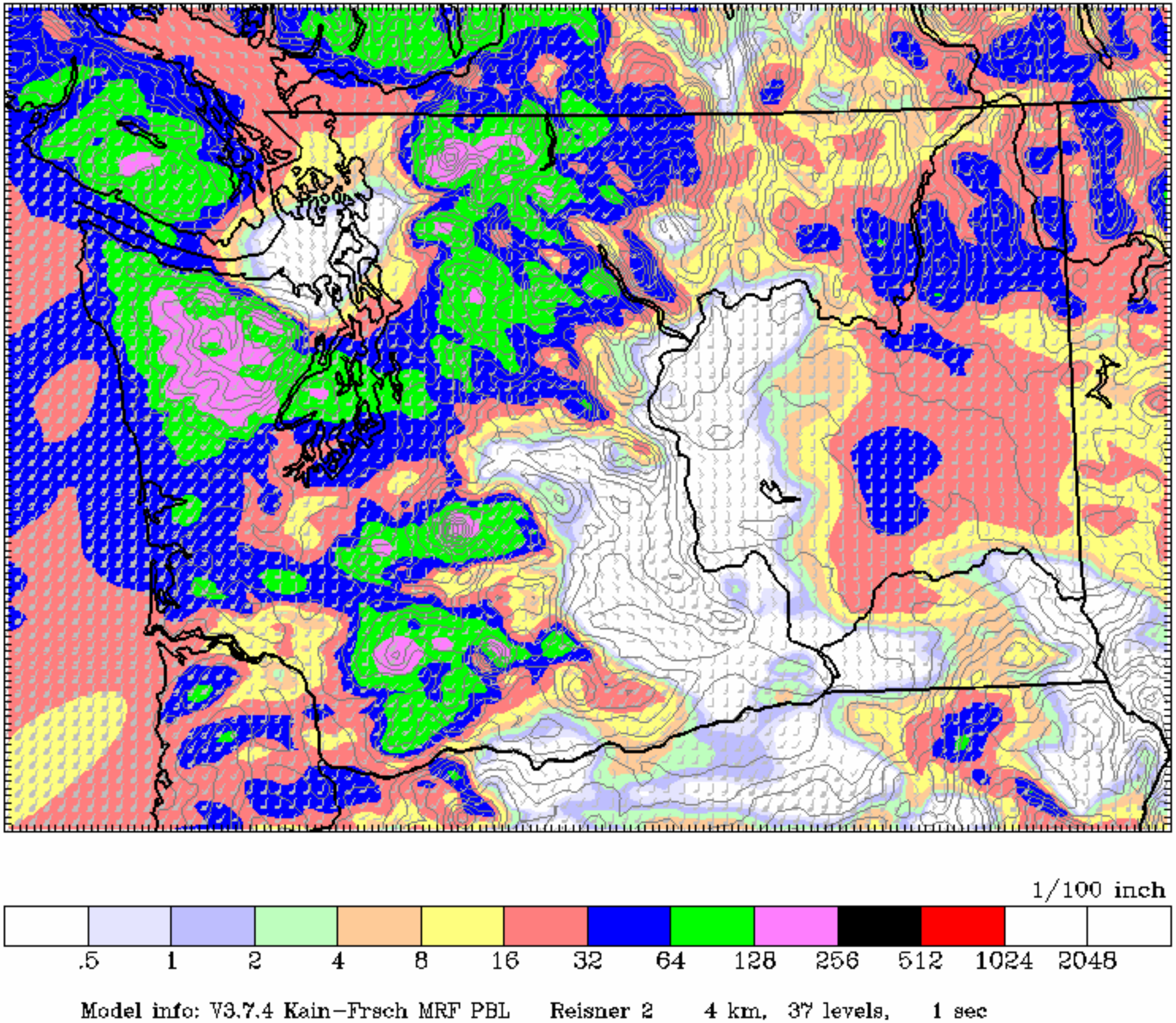


NGM, 80 km, 1995

1995



2007-2008

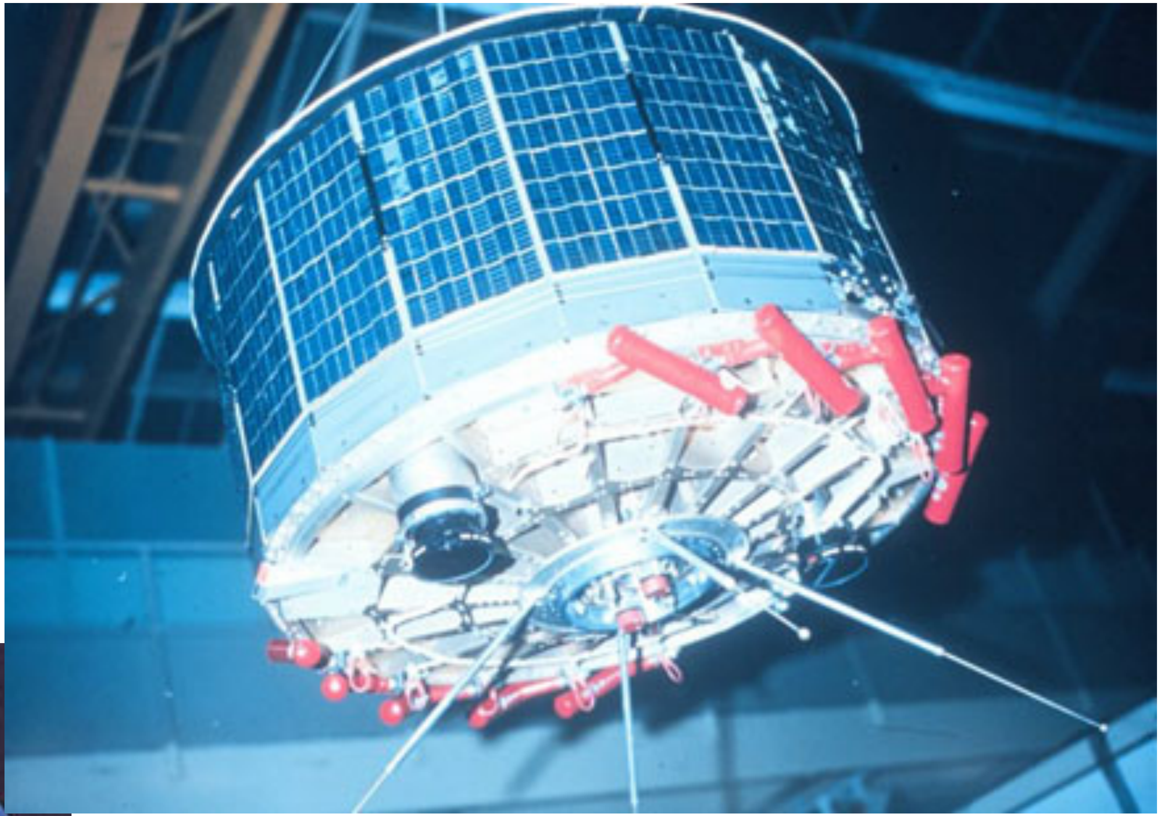


4-km grid spacing

But there was another revolution that drove numerical weather prediction… in weather observations, with weather satellites taking the lead

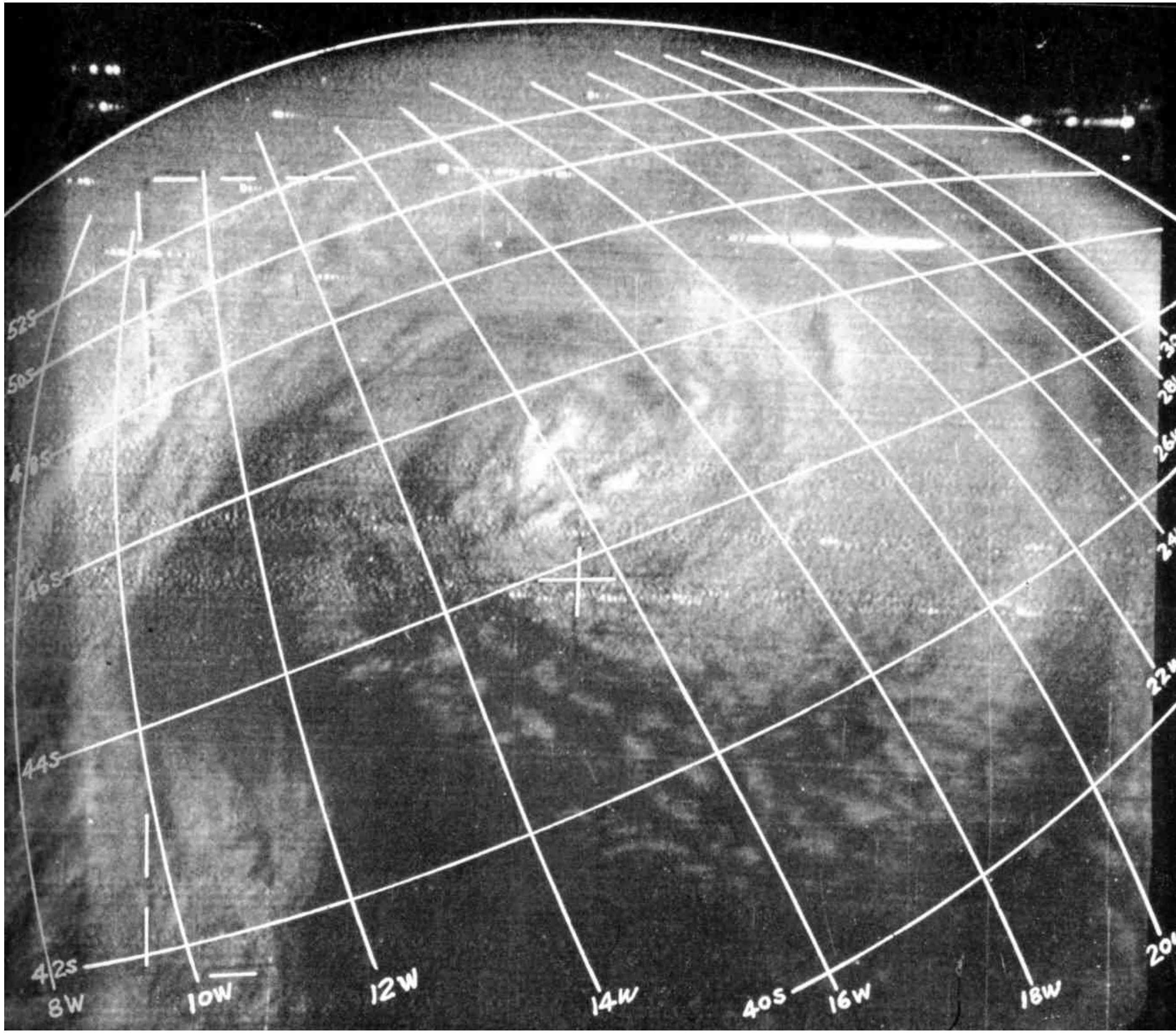
- There was little data over most of the planet (oceans, polar regions)
- Storms could approach the coast without warning and numerical modeling was crippled by lack of data.

1938 Hurricane



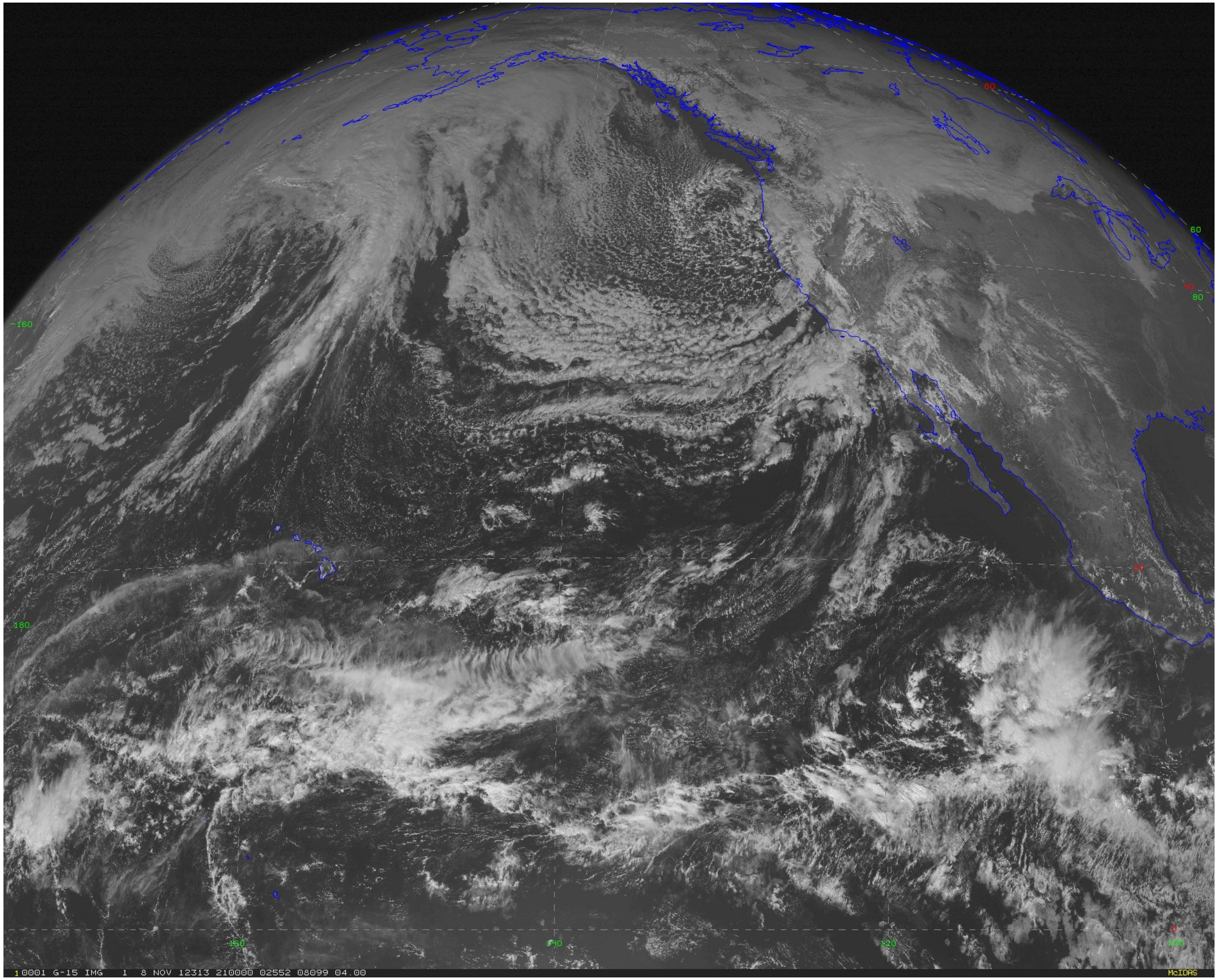
TIROS-1: The First Weather Satellite (polar orbiter)--1960

First Weather Satellite Image

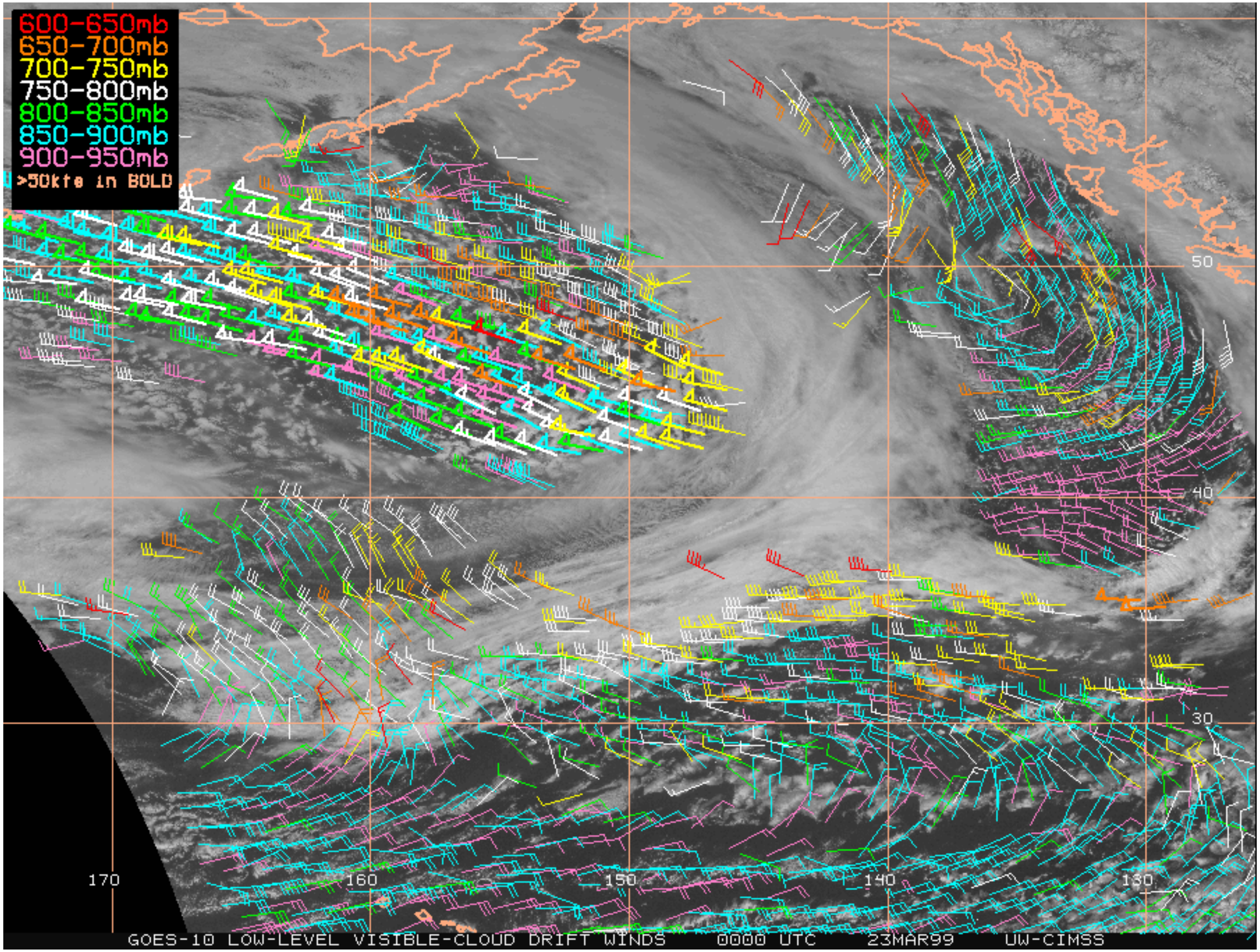


Starting in 1974, NOAA GOES (Geostationary) Satellite

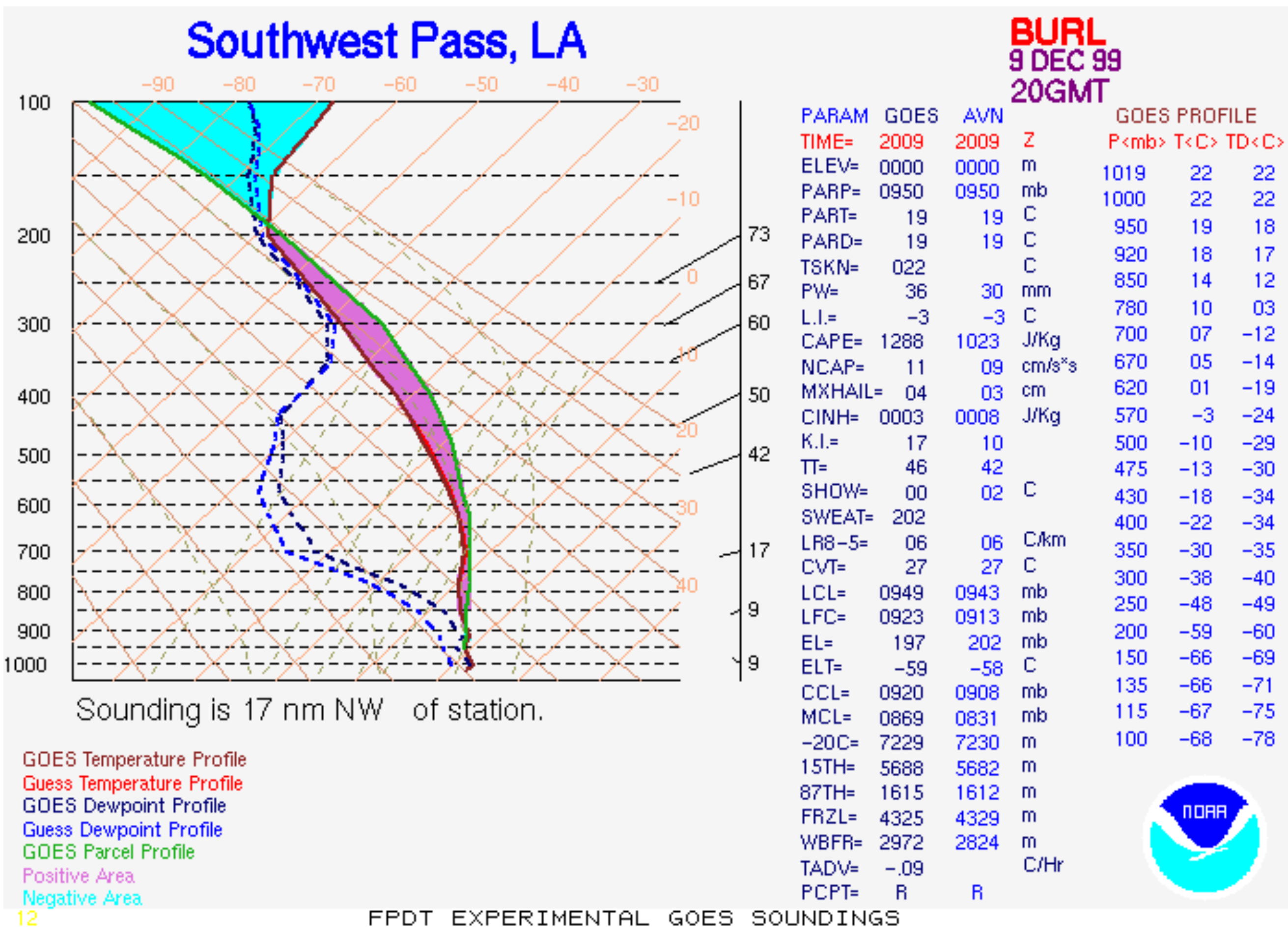
Weather Satellites Now View the Entire Planet

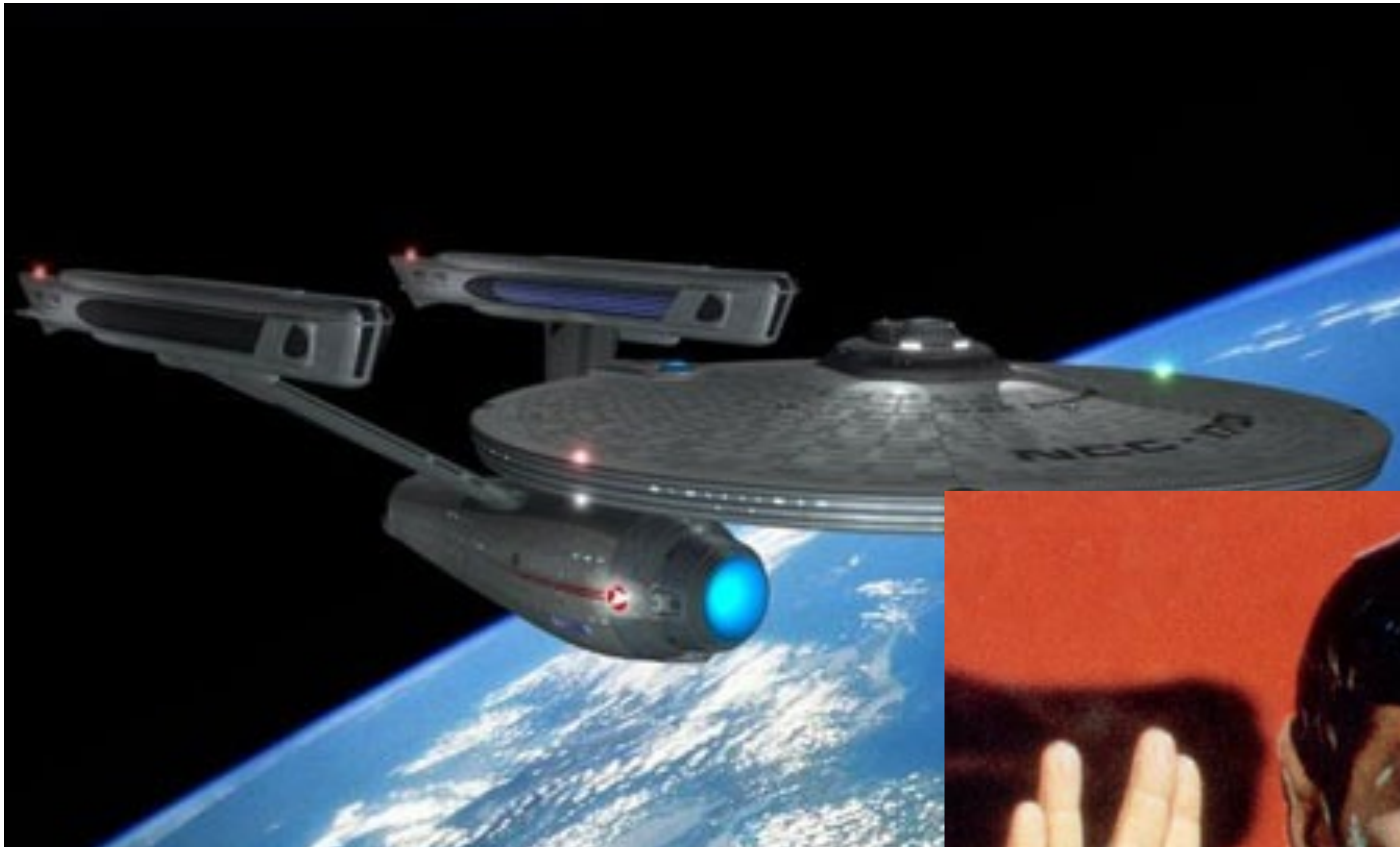


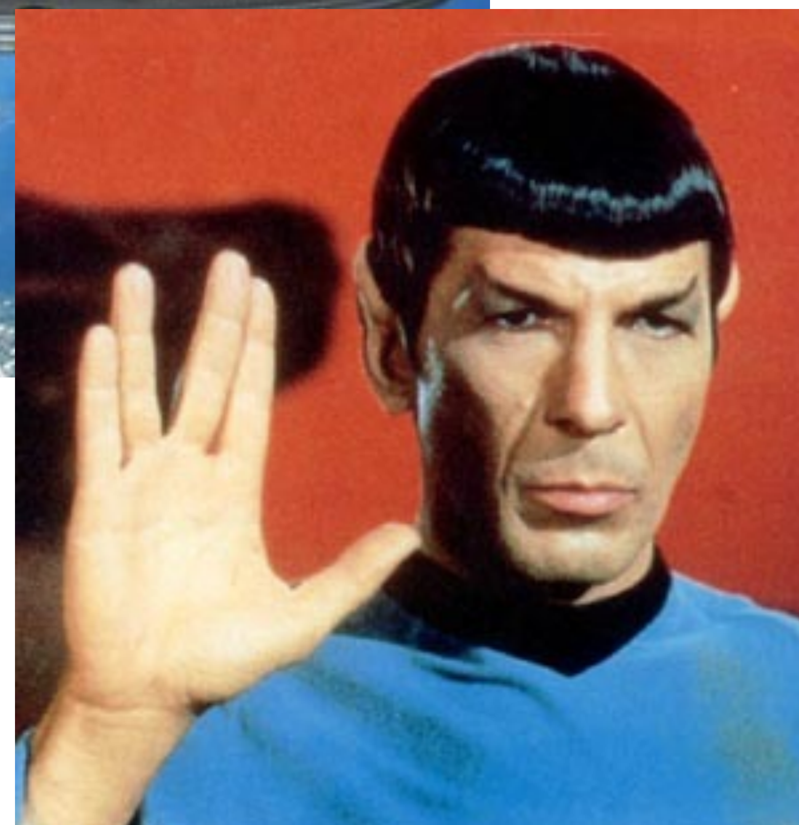
Cloud Track Winds



Satellite Sensors Provide Thousands of High Quality Vertical Soundings Daily over the Pacific

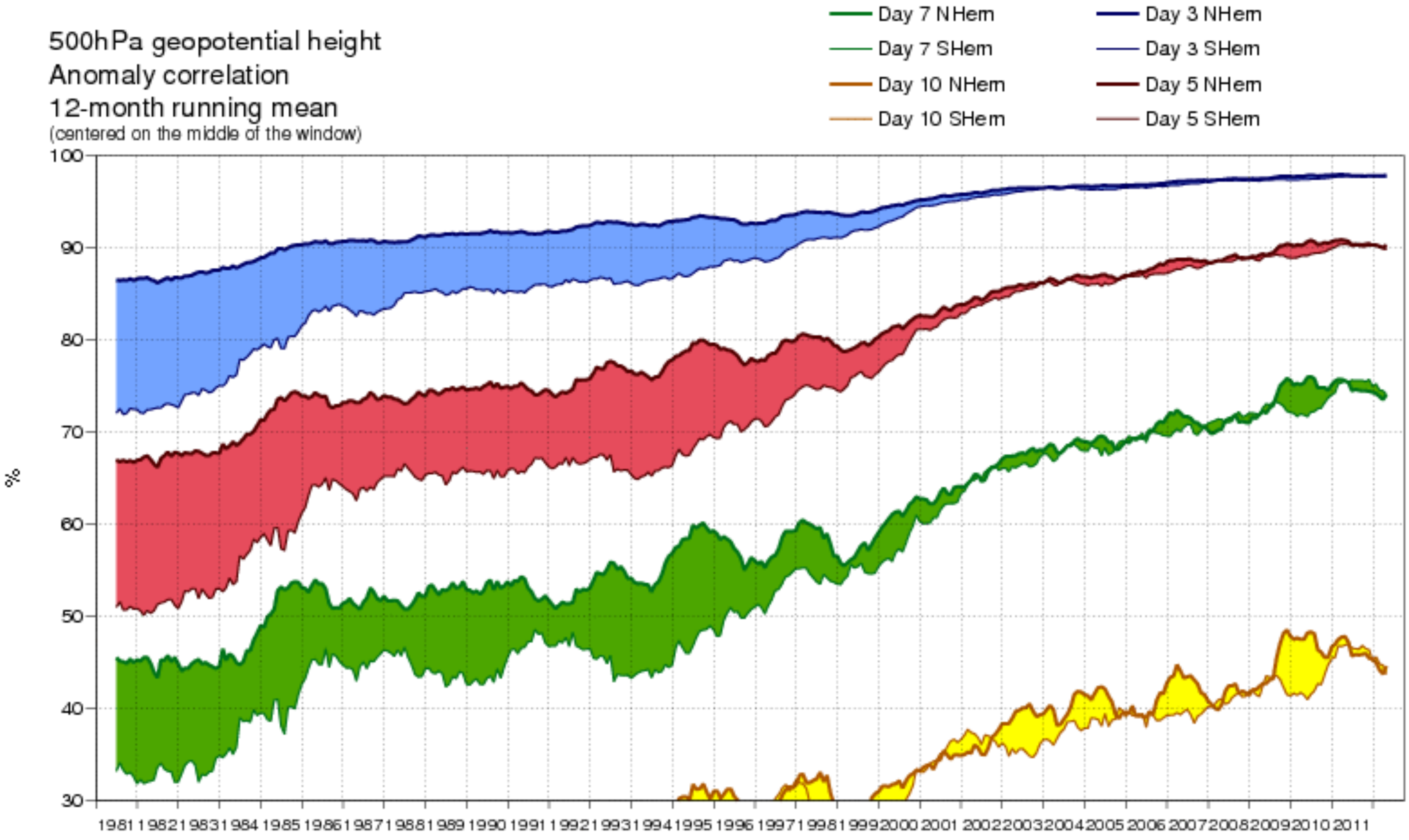






Better than Star Trek!

Because of weather satellites, forecast skill is the same in the northern and southern hemispheres



A Fundamental Problem

- The way we have been forecasting has been **essentially flawed.**
- The atmosphere is a chaotic system, in which small uncertainties in how we start the forecast (the ***initialization***) have large impacts on the forecasts.
- Not unlike a pinball game….

A Fundamental Issue

Mathematician/meteorologist Ed Lorenz found that small errors in initial conditions can grow so that **forecast skill is lost at about two weeks**.

Butterfly Effect: a small change at one place in a complex system can have large effects everywhere over time

A Fundamental Problem

- Similarly, there are uncertainties in processes, like the development of clouds and precipitation, which produce uncertainty in forecasts.
- **Thus, all forecasts have uncertainty.**
- The uncertainty generally increases in time.

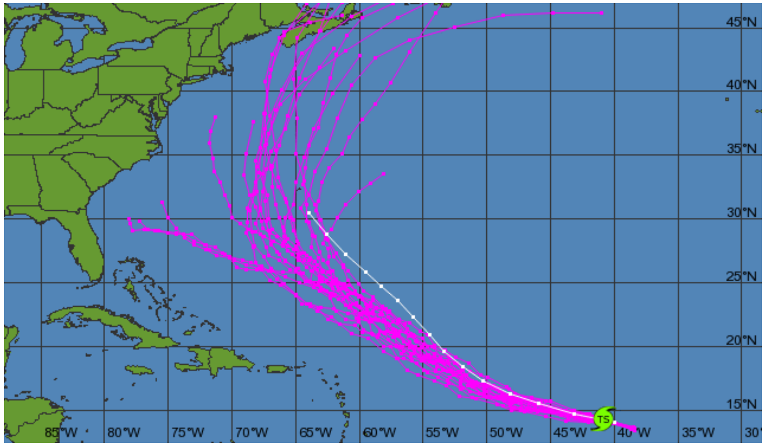
This is Ridiculous!

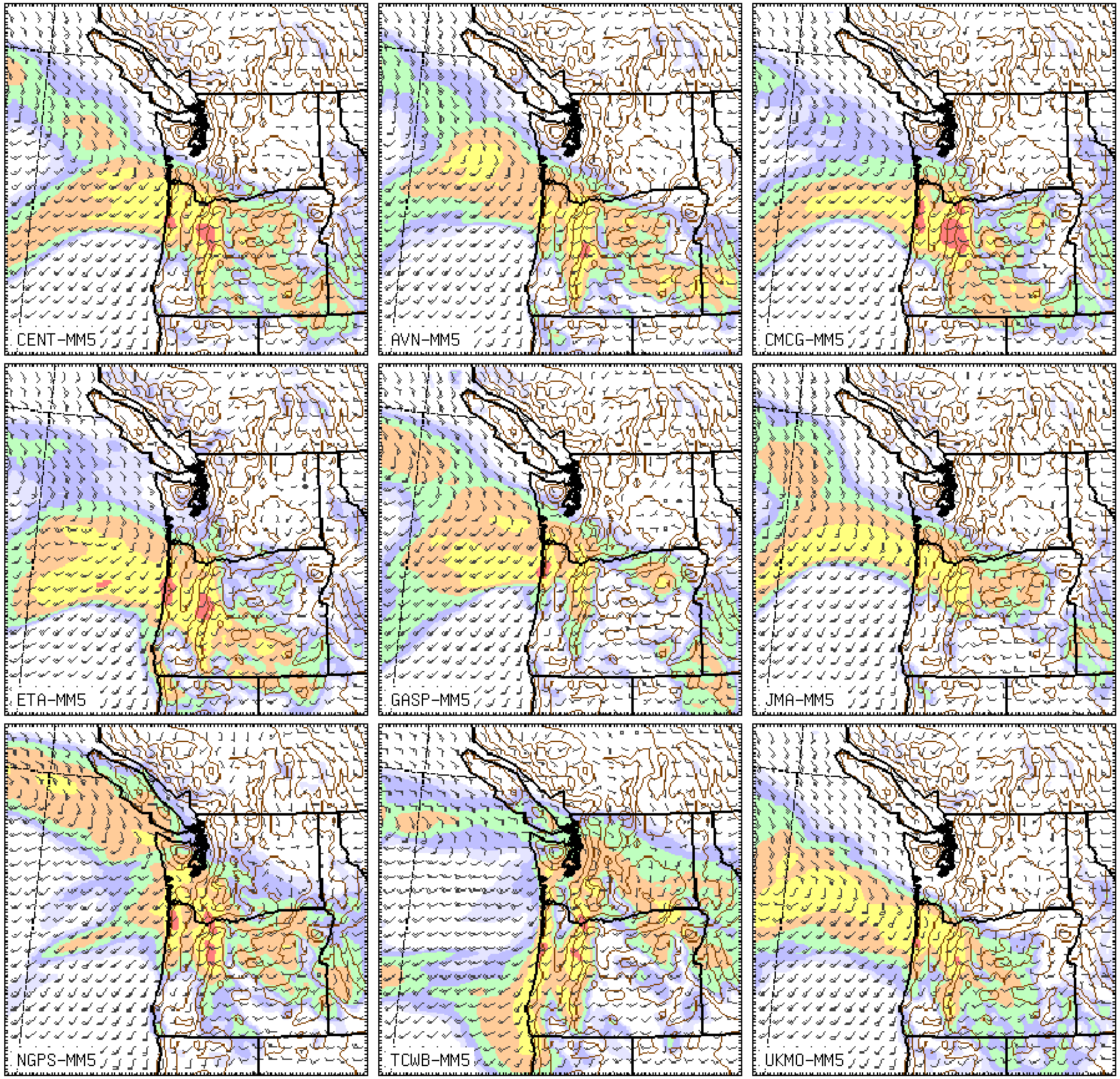
We Must Forecast Probabilistically

- **Weather forecasters need to tell users about the uncertainties in the forecast**.
- Give forecasts in terms of probabilities.
- There is an approach to produce such guidance …ensemble forecasts

Ensemble Prediction

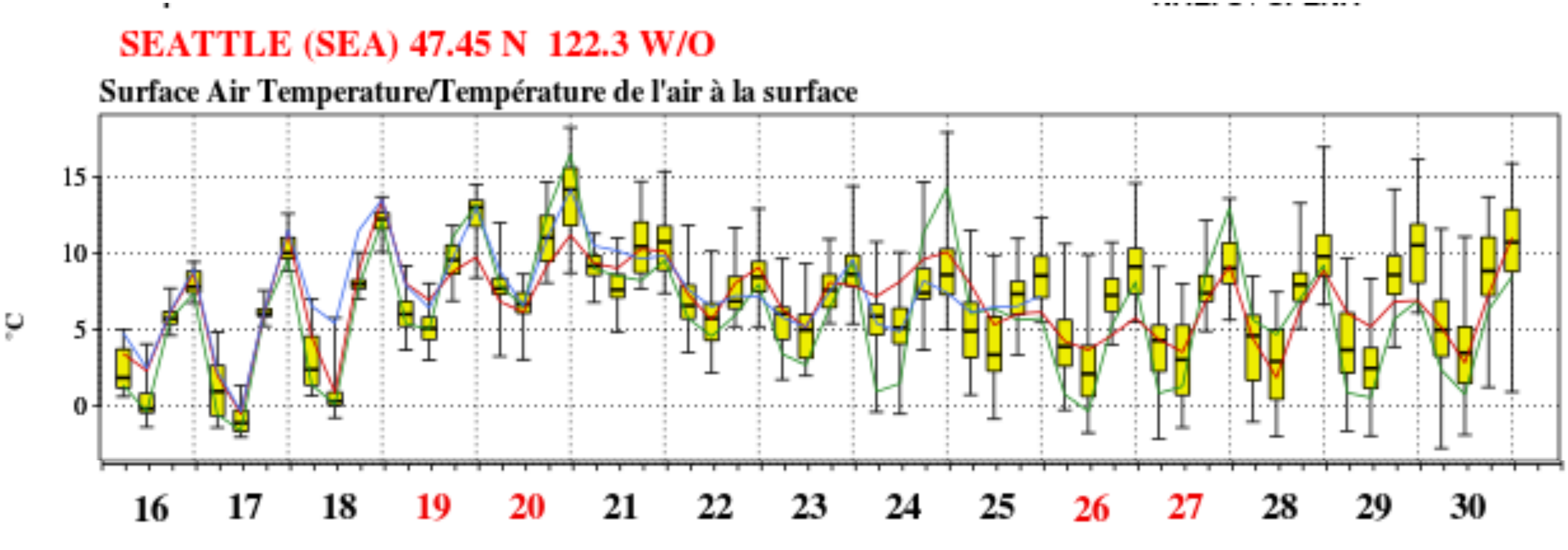
- Instead of making one forecast…make many…each with a slightly different initializations or different model physics.
- Possible to do this now with the vastly greater computation resources that are available.




CENT-MM5
AVN-MM5
CMCG-MM5
ETA-MM5
GASP-MM5
JMA-MM5
NGPS-MM5
TCWB-MM5
UKMO-MM5

Ensemble Prediction

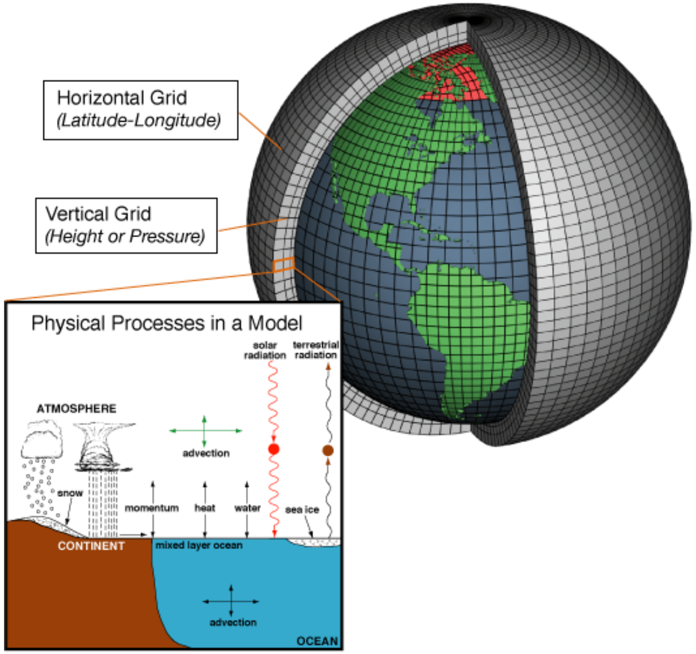
- Ensemble can give probabilities.
- The **ensemble mean or average** is more accurate than any individual member.
- Ensembles will dominate forecasting in the future.



What is a forecast model?

Not this!

A forecast model includes the equations that describe the physics of the atmosphere, the mathematical techniques used to solve the equations, and the data assimilation techniques used to prepare the initialization from /observations



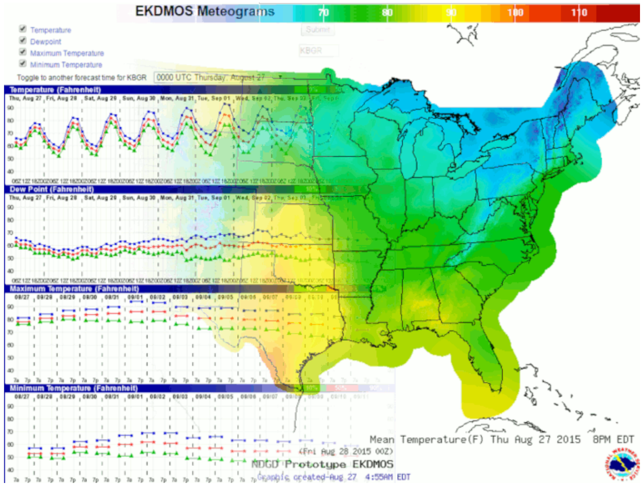
Numerical Weather Prediction Uses Some of the Biggest Computers on the Planet!

- Our forecast models use the largest supercomputers and run on many processors simultaneously.
- This requires very fast code that is parallelized to do many things quickly.

Cray XC40

Even More Mathematics In Weather Prediction!

- Numerical weather prediction produces a future description of the atmosphere for all atmospheric variables on a three-dimensional grid of points.
- **We can apply statistical techniques to improve the forecasts further.**
- How? We can correct for known past biases and errors.
- Called **post-processing.**



Online products from groups such as the weather company, IBM, Accuweather and others—which drive your smartphone weather apps—are very good at such statistical post-processing

Weather forecast accuracy for Seattle, Washington

Last Month

The Weather Channel	**89.00%**
Weather Underground	**87.31%**
Dark Sky	**82.81%**
Foreca	82.25%
AccuWeather	81.70%
NWS Digital Forecast	78.89%
World Weather Online	74.60%
Persistence	52.21%

Last Year

The Weather Channel	**85.72%**
AccuWeather	**84.49%**
Weather Underground	**83.34%**
Foreca	82.27%
MeteoGroup	82.20%
NWS Digital Forecast	77.10%
Dark Sky	76.56%
World Weather Online	67.06%
Persistence	55.35%

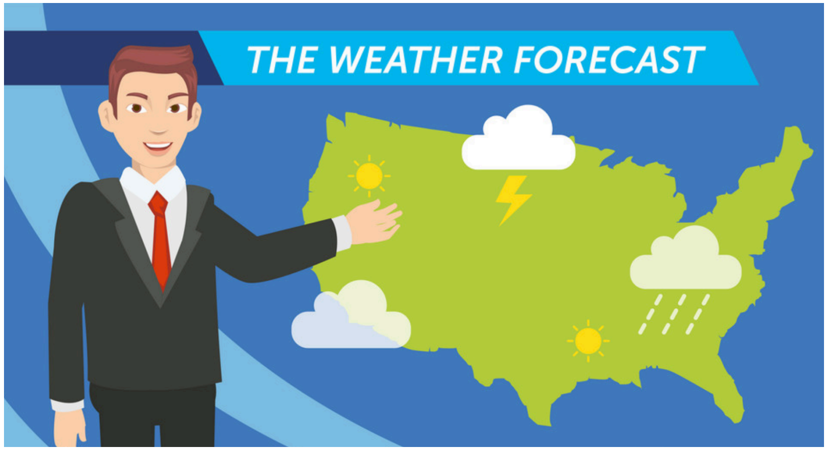
Machine learning approaches to improving numerical weather forecasting are being rapidly developed

Numerical Weather Prediction is the Biggest of Big Data

- Huge amounts of data coming in each from weather satellite, radars, and other observing platforms. (petabytes per day)
- Huge amount of output from numerical weather prediction models.
- Moving and storing all this data is very difficult.
- Making use of this huge data stream is great challenge

The Bottom Line

- Numerical Weather Prediction may be the complex and cooperative mathematical activity of our species
- Involves solving a complex series of partial differential equations and highly mathematical data assimilation of observations.
- Requires worldwide cooperation of gathering observations from weather satellite and other assets worth tens of billions of dollars.
- Requires the use of the largest supercomputers
- Involved sophisticate statistical post-processing of model forecasts.
- And there is much more progress to be made!



The end