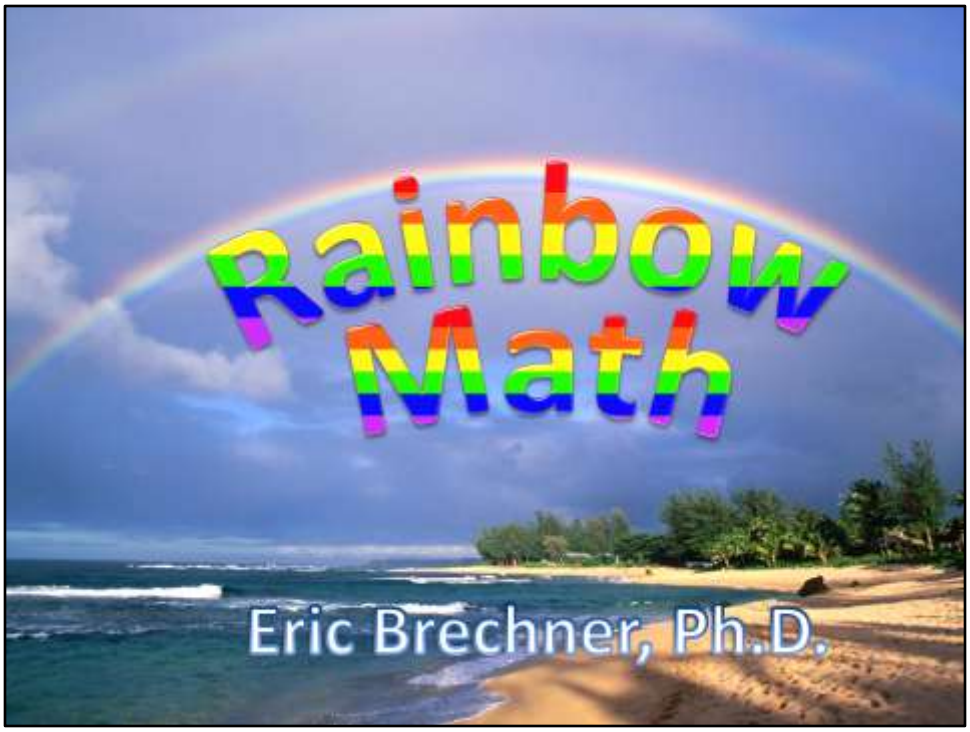
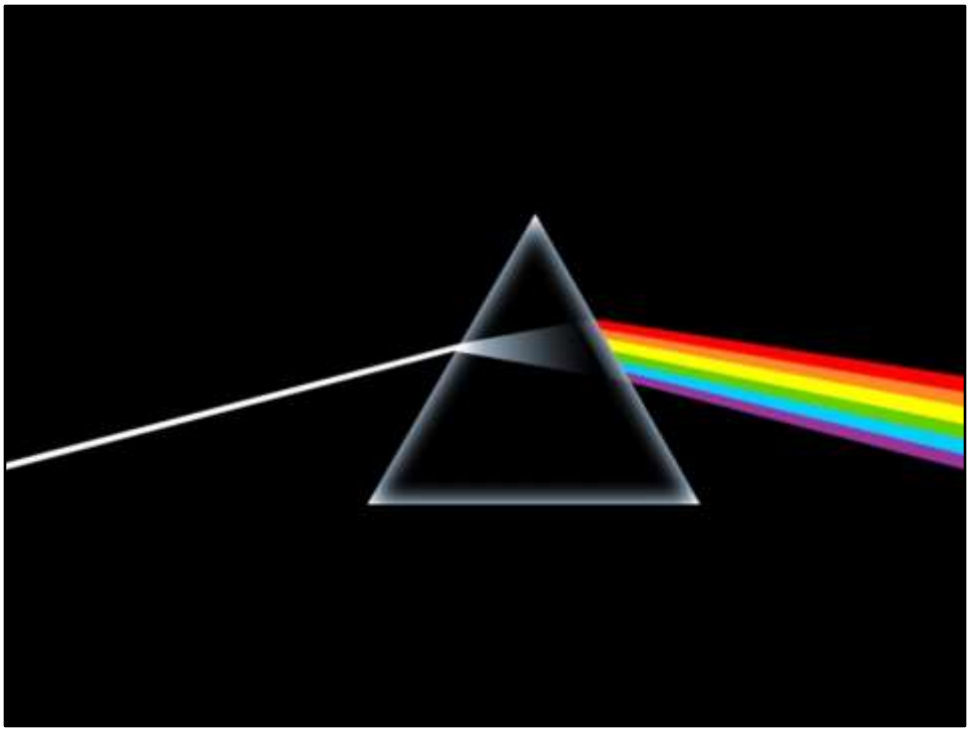


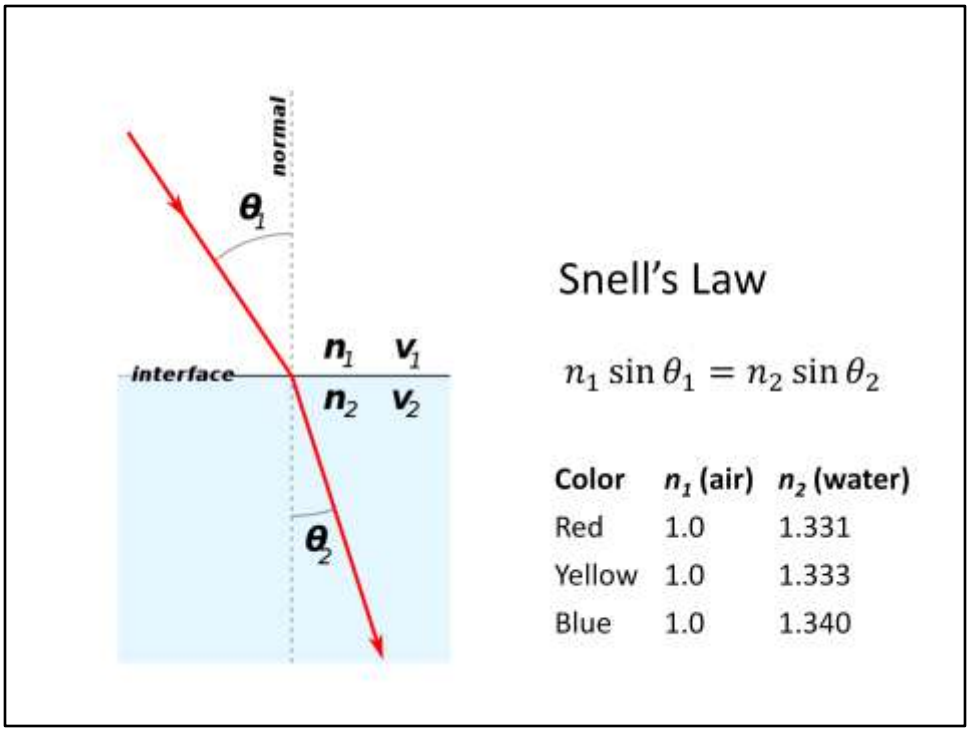
What time of day is best to see a rainbow? (Morning, Noon, Evening, Night)
What direction should you look? (N, S, E, W)
Which color is on top? (Blue, Yellow, Green, Red)
Why is a rainbow shaped like an arch? (Sun's shape, Earth's shape, Water's shape, Sun's distance)
Are there ever two rainbows at once? (Yes, in different directions; Yes, in the same direction; No)

http://thuvienbao.com/anhdep/wp-content/original/2009_12/Rainbow-10.jpg

May 13, 2012
Location: Room 260, Savery Hall
Speaker: Eric Brechner, Principal Development Manager, Xbox Engineering Fundamentals
Title: "Rainbow Mathematics"
Abstract: What time of day is best to see a rainbow? Why is a rainbow shaped like an arch? Which color is on top? Are there ever two rainbows at once? Rainbows are uncommonly beautiful. Most people have seen them, especially here in Seattle. Yet, most people don't know a rainbow's secrets. A little optics, some math, and your imagination are all you need to unlock rainbows and reveal things few people know. You'll uncover them all for yourself in this engaging talk that turns Snell's law, water, sunlight, and reflection into a beautiful sight.

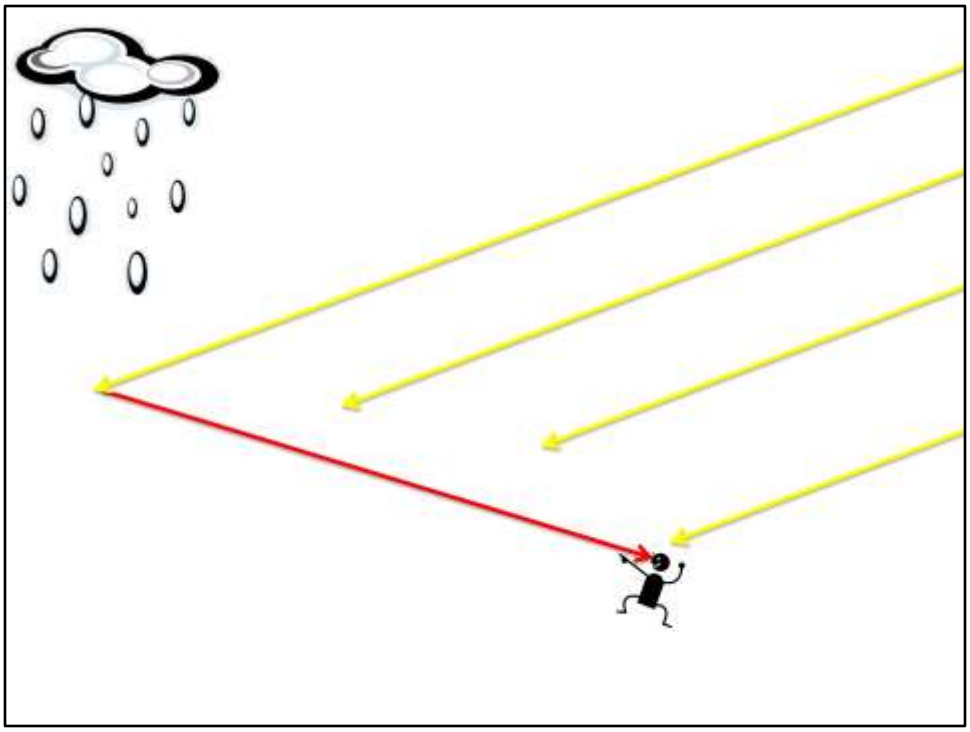


What is this a picture of? (album cover, prism)
Key to understanding rainbows is the math for the bend
(Tricky equations, so we'll talk about them first)

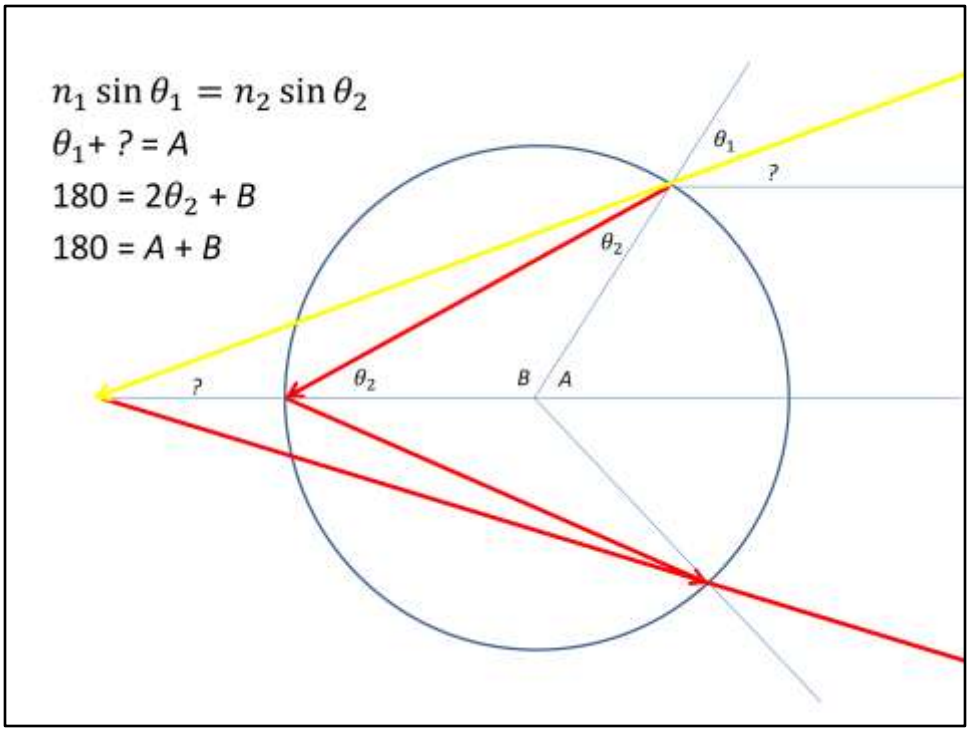


http://en.wikipedia.org/wiki/Snell%27s_law
The "normal" is the line perpendicular to the interface between the air and water.
n = refraction index (we know n1 and n2!!!)
v = phase velocity
For us to get the colors of a rainbow the light must go through the water.

http://thuvienbao.com/anhdep/wp-content/original/2009_12/Rainbow-10.jpg
Where is the sun?
Notice the shadows.



We want to know the angle that gives us red. It should be different for yellow and blue.
Note that light from the sun is hitting everything (including you) at the same angle, because the sun is so far away.
Let's take a close look at the raindrop.



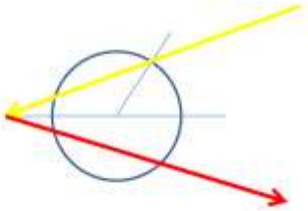
1. What is the shape of a raindrop? (circle)
2. We need normals to the raindrop. That's easy—they are lines from the center.
3. Now we can see how the light bends, reflects off the back, and then bends again leaving the raindrop
4. So, we've got our angles and Snell's law
5. We want the ? angle. Why is it the same in both spots? (parallel lines)
6. What about angle A? What does it match?
7. What about angle B?
    - Why is theta2 the same in both spots? (Isosceles triangle)
    - What do we know about angle B and the two theta2 angles?
8. What about angles A and B? What do they add up to?
9. Four equations.
    - What values do we know? (n1, n2, theta1)
    - What values do we not know? (A, B, theta2, ?)
    - Four equations, four unknowns. Yay! We can solve it!

$n_1 \sin\theta_1 = n_2 \sin\theta_2$

$\theta_1 + ? = A$

$180 = 2\theta_2 + B$

$180 = A + B$

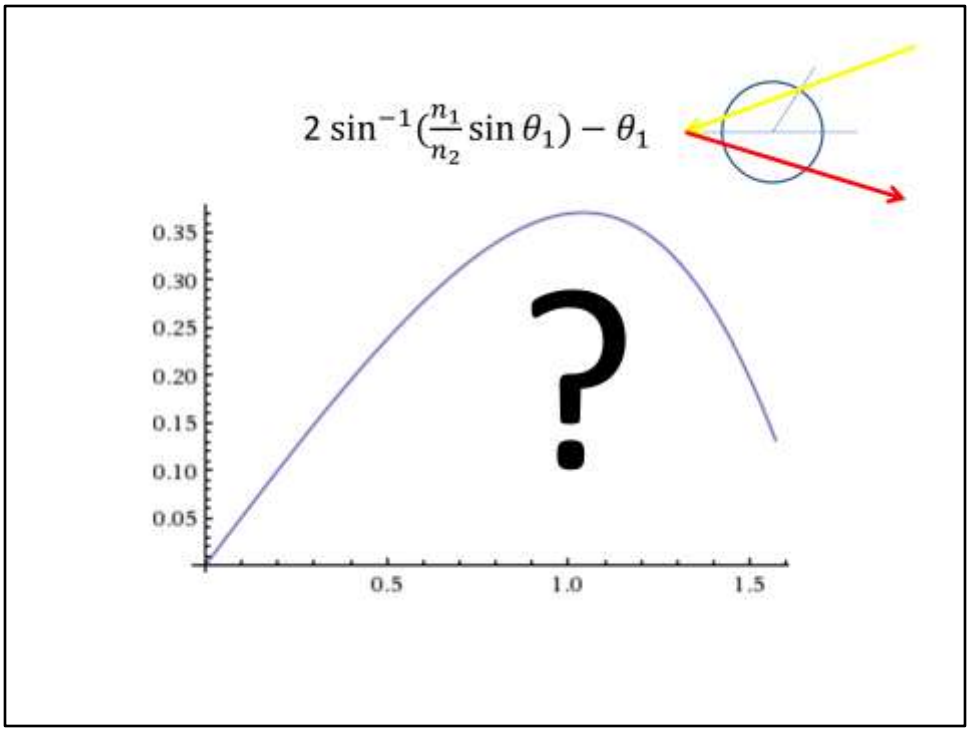


$\theta_1 + ? = 2\theta_2$

$? = 2\theta_2 - \theta_1$

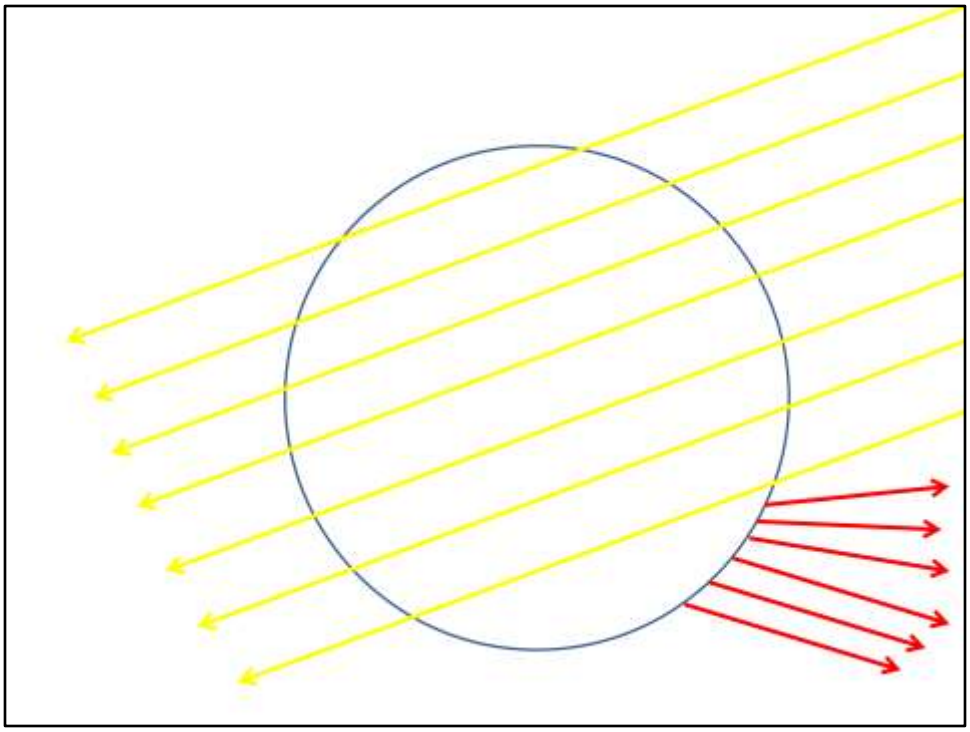
$$? = 2\sin^{-1}\left(\frac{n_1}{n_2}\sin\theta_1\right) - \theta_1$$

1. We know 2 theta2 is the same as A
2. We can solve for ?
    - We can solve for theta2 and plug it in (complicated, but not bad)
3. We've done it! But it looks nasty. What now? Let's plot it!

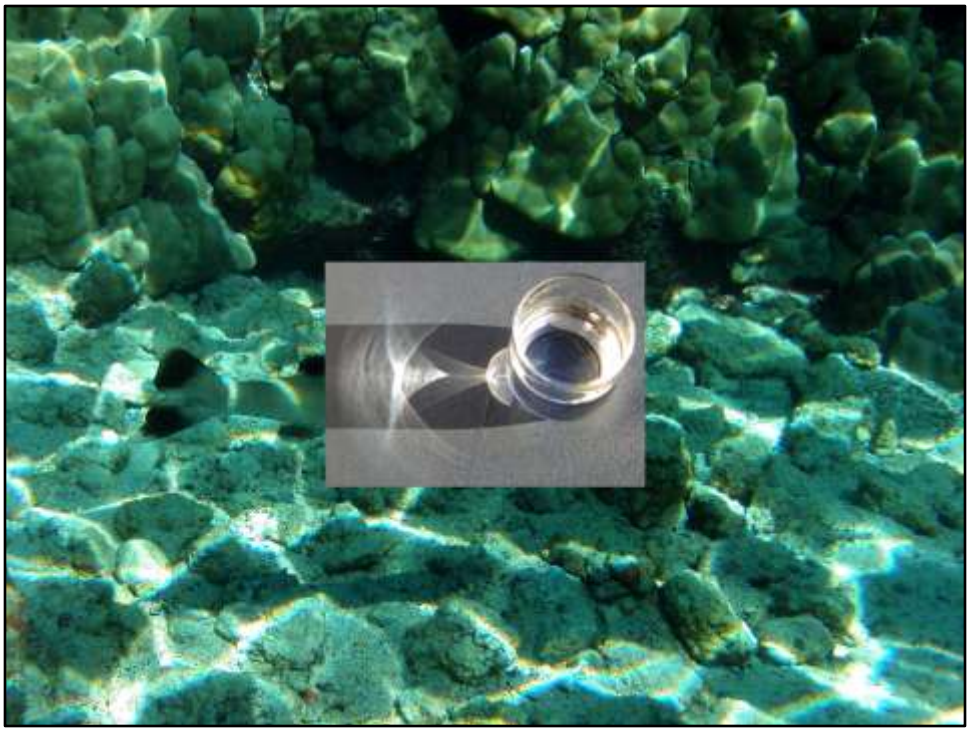


http://www.wolframalpha.com/input/?i=2*asin%28%281%2F1.33%29*sin%28x%29%29+-+x+from+x+%3D+0..pi%2F2

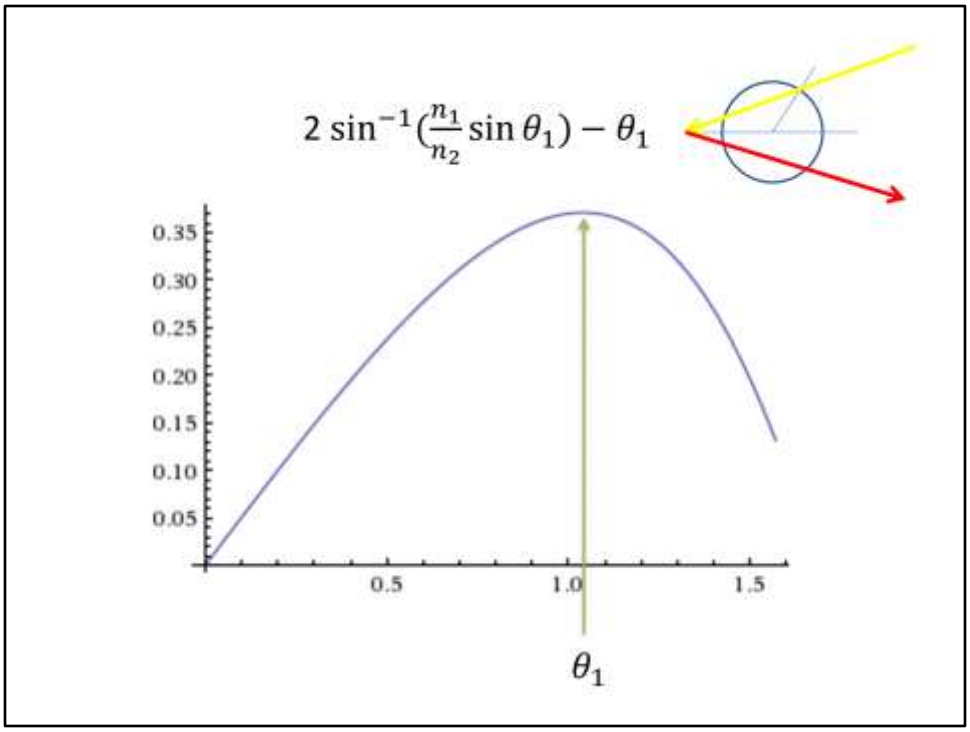
? is small for small theta1, stays big in the middle, and small for large theta1



Let's see what that looks like on the circle (not precise).
Some of the sunlight goes out at different angles, but goes out at the same angle.



http://en.wikipedia.org/wiki/Caustic_(optics)
Here's what it looks like when light gets focused through water.
They are called caustics.
You've seen them at the aquarium too.

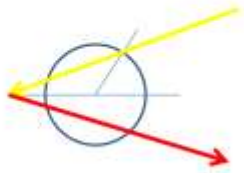


http://www.wolframalpha.com/input/?i=2*asin%28%281%2F1.33%29*sin%28x%29%29+-+x+from+x+%3D+0..pi%2F2

This is the angle we want with a concentration of color!

The one at the top in the middle where the angle stays the same and the light gets focused.

$$\theta_1 = \cos^{-1}\sqrt{\frac{(\frac{n_2}{n_1})^2 - 1}{3}}$$



$$? = 2\sin^{-1}(\frac{n_1}{n_2}\sin\theta_1) - \theta_1$$

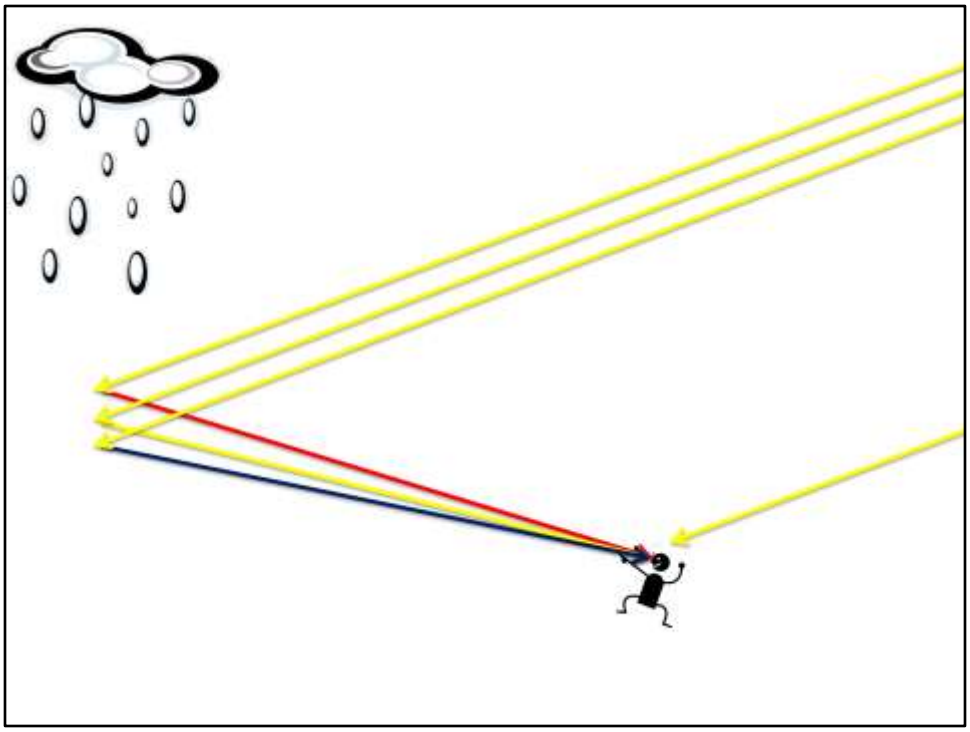
Color	n_1 (air)	n_2 (water)	θ_1	?
Red	1.0	1.331	59.53	21.18
Yellow	1.0	1.333	59.41	21.04
Blue	1.0	1.340	59.00	20.54

http://www.wolframalpha.com/input/?i=solve+d%2Fdx+%282*asin%28%28n1%2Fn2%29*sin%28x%29%29+-+x%29+%3D+0+for+x

Using some calculus and more Wolfram Alpha, you can find theta1.

Once you have theta1, you can find ? given n1 and n2.

The red angle is the biggest. The yellow angle is smaller. The blue angle is the smallest.



1. We know the red angle.
2. The yellow angle is smaller and appears lower.
3. The blue angle is smallest and appears at the bottom.

Red is on top!

Let's go back over the questions.

- What time of day is best to see a rainbow? Morning, Evening
- What direction should you look? E, W
- Which color is on top? Red
- **Why is a rainbow shaped like an arch?** (use pipe cleaner) Water's shape, Sun's distance

http://thuvienbao.com/anhdep/wp-content/original/2009_12/Rainbow-10.jpg
Are there ever two rainbows at once? Yes, in the same direction.
Look above. The extra bounces within the raindrop make it dimmer and invert the color order.

http://2pat.files.wordpress.com/2010/05/double-rainbow.jpg
For the second rainbow, the red is at the bottom and the blue is at the top.