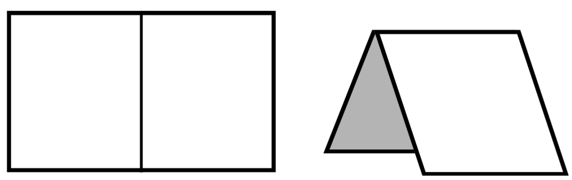
UW Math Circle

Week 15

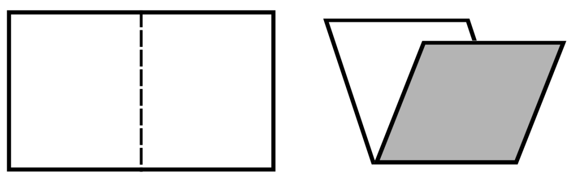
Origami is the art of folding papers into different shapes. Some origami models can be pressed flat and be stored in a book, while others cannot. If an origami model can be pressed flat without creating any additional creases, we call it *flat-foldable*. Today, we will study which origami models can be pressed flat, and which ones cannot.

Crease Pattern

A *crease pattern* is a diagram showing all creases(folding lines) of an origami model on a single sheet of paper. A *valley fold* creates a V-shape ”trench”, represented by a dashed line on crease patterns in this worksheet. A *mountain fold* creates a ”peak”, represented by a solid line on crease patterns in this worksheet.

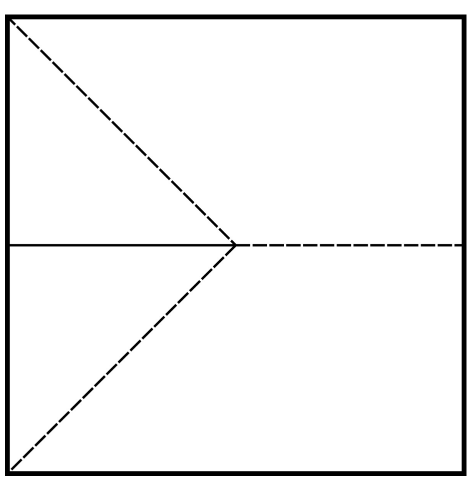


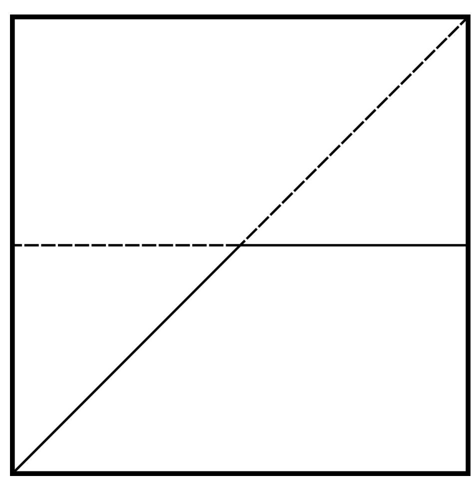
Mountain Fold



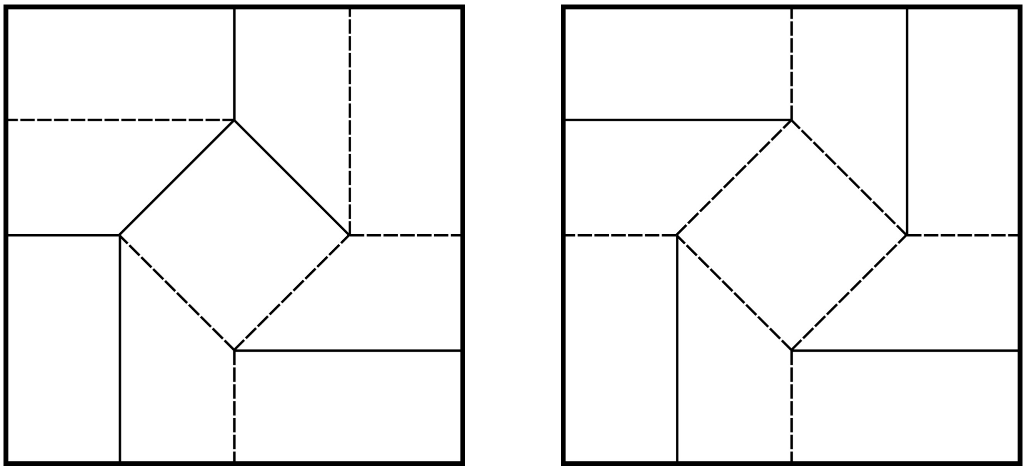
Valley Fold

For example, the diagram on the left is the crease pattern for a flat foldable origami model, and the diagram on the right is the crease pattern for a not flat foldable origami model. Ask a instructor to show you the origami models for these crease patterns.





1. Two crease patterns with the same position of lines can lead to different results. Work with your teammate to explore the two crease patterns below. Each of you will choose a different pattern from your teammate and fold your paper following your chosen crease pattern. Compare your results with your teammate.

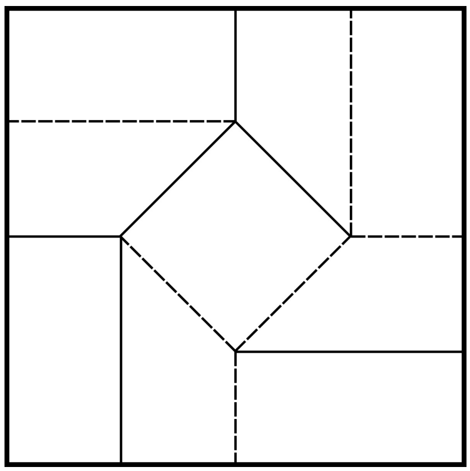


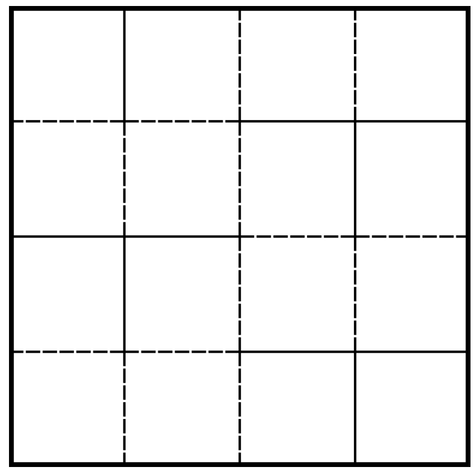
2. Fold your origami paper into something that is flat, and record down your own flat-foldable crease pattern. Show an instructor that the pattern is flat-foldable.

Stop here. Request the next page from your instructor when your group is done.

Mountains and Valleys

3. We call vertices that are not on the boundary of the paper *interior vertices*. Both crease patterns shown below are flat foldable. For each of them, at each interior vertex, count the number of mountain folds and the number of valley folds. What is the difference between these two numbers for each vertex?

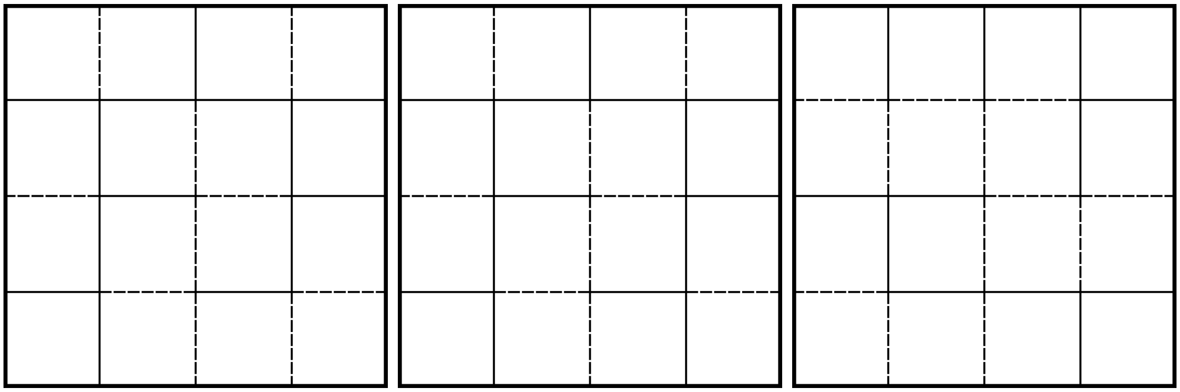




4. Check your own crease pattern on page 2. How many interior vertices does it have? What is the difference between the number of mountain folds and valleys folds at each interior vertex?

5. Based on your observation, what can you conjecture about the difference between the number of mountain-fold lines and the number of valley fold lines at each interior vertex on a flat-foldable pattern?

6. Using your answer from the previous problem, which one(s) of the following crease patterns do you think are not flat-foldable, why?

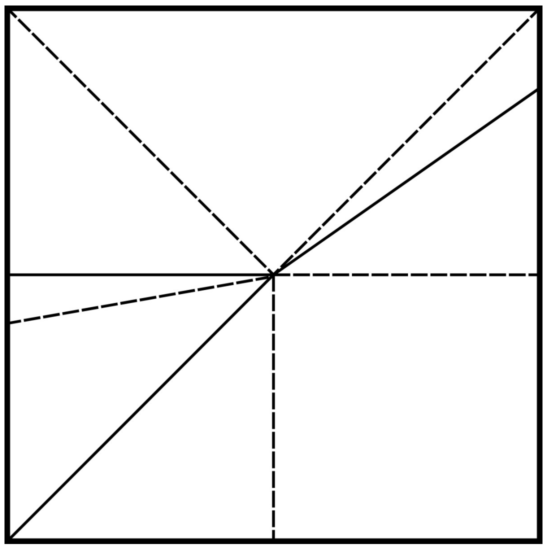


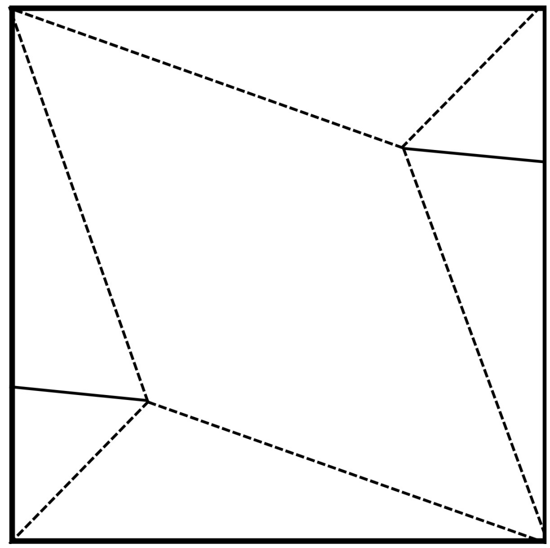
7. (Challenge) Can you come up with a crease pattern, where the numbers of valley and mountain folds differ by two at every interior vertex, but still not flat-foldable?

Stop here. Request the next page from your instructor when your group is done.

Even and Odd Angles

8. The two patterns given below are both flat foldable. (Optional: Ask an instructor to show you that they are indeed flat foldable)





At each interior vertex, the creases creates many angles. Follow the instructors below on the two flat foldable crease patterns:

(a) Starting at any angle, number the angles around the vertex in order clockwise.

(b) Measure each angle with a protractor.

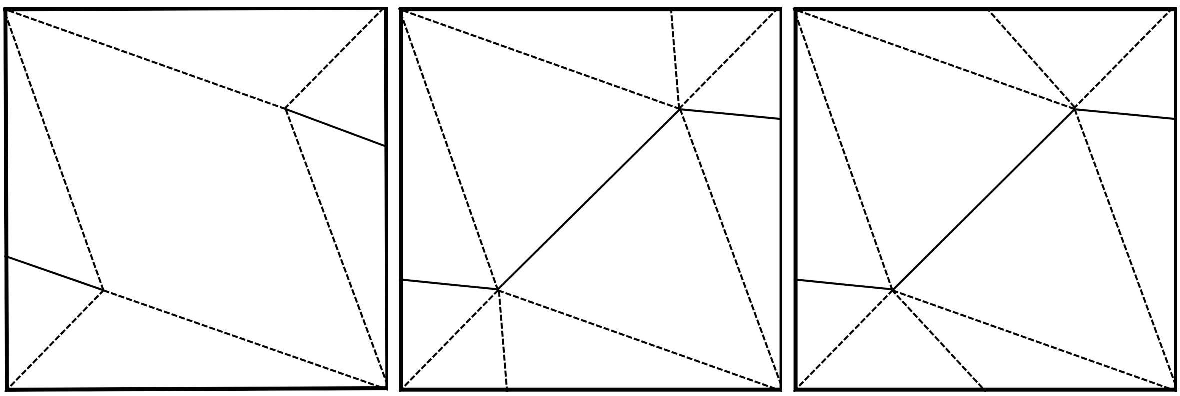
(c) Add up all the odd-numbered angles, what do you get?

(d) Add up all the even-numbered angles, what do you get?

9. Try this with one interior vertex on your own crease pattern on page 2.

10. Based on your observation, what can you conjecture about the sum of odd-numbered angles and the sum of even-numbered angles at an interior vertex for a flat foldable crease pattern?

11. Using your answer from the previous problem, which ones(s) of the following crease patterns are not flat-foldable, why?



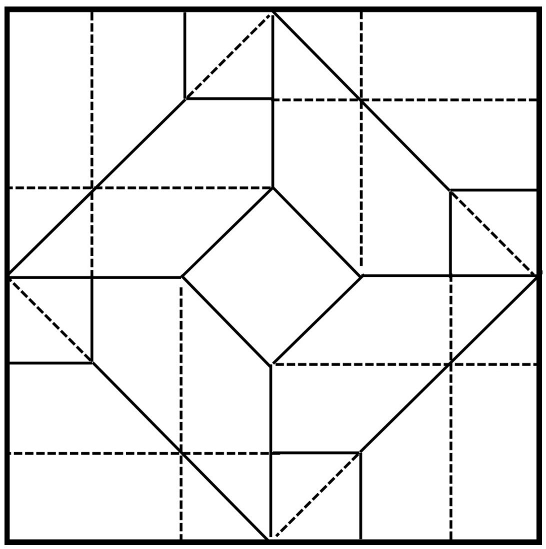
12. Does it matter which angle you picked as Angle 1? why? Does it matter if you label them clockwise or counterclockwise? (Hint: what does your answer to problem 5 imply about the number of creases at a vertex?)

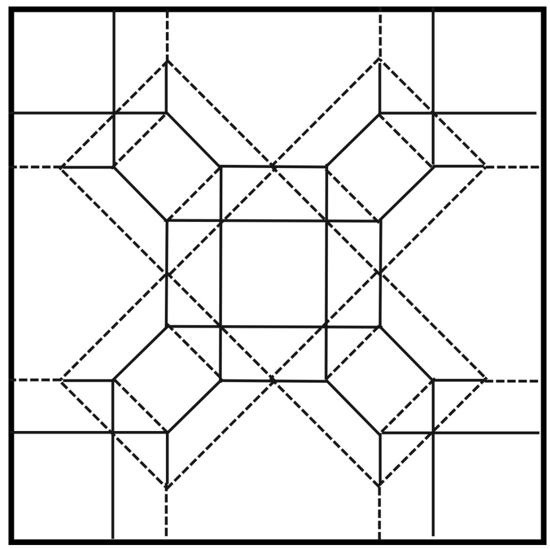
Stop here. Request the next page from your instructor when your group is done.

Coloring the Regions

Recall from the graph theory worksheet, a proper coloring of a map is a coloring where no two bordering regions share the same color. Now, we will color the regions on a crease pattern similarly to maps.

13. Both of the following two crease patterns are flat foldable. What is the minimum number of different colors necessary to properly color each of them?





14. What is the minimum number of different colors necessary to properly color your own crease pattern from page 2?

15. What can you conjecture about the minimum number of colors necessary to properly color a flat foldable crease pattern?

16. Can you come up with crease pattern that satisfies the Mountain and Valley condition, but fail the coloring condition? If you think this is impossible, explain your reasons.

Stop here. Request the next page from your instructor when your group is done.

17. Create your own not flat foldable crease pattern.

18. Are the three conditions discussed today sufficient for a crease pattern to be flat foldable? That is, if a crease pattern meets all three conditions, is it guaranteed to be flat foldable? If your answer is no, give an example of a crease pattern that satisfies all three conditions, but is not flat foldable. If your answer is yes, explain your reasons.

19. Justify your conjectures from problem 5, 10, and 15.