UW Math Circle

Week 12

Creating Graphs

Recall the *degree* of a vertex is the number of edges connected to that vertex.

Use pom-poms as vertices and string as edges to help answer the following problems:

1. Is it possible to create a graph with 6 vertices so that every vertex has degree 4? Why or why not?

2. Is it possible to create a graph with 5 vertices so that every vertex has degree 3? Why or why not?

3. Starting with 7 vertices, can you create a graph in which every vertex has a different degree? Why or why not?

4. What number of starting vertices allow creating a graph so that every vertex has degree 4? What about degree 3? What about degree k?

Three Utilities Problem

1. Find the paper with pictures of three houses and three utility centers. Is it possible to use string to connect each house to each utility center so that no two strings overlap? Why or why not?

2. Next find the tissue box with three houses and three utility centers. You are now allowed to wrap the string around the box so long as the string is always making contact with the box. Now is it possible to connect each house to each utility center so that no two strings overlap? Why or why not?

3. Next find the cardboard tube. Keeping each string in contact with the tube, is it possible to connect each house to each utility center so that no two strings overlap? Why or why not?

4. Suppose we want to connect 4 houses to 4 utility centers so that no two lines overlap. Is there a shape that makes this possible? Why or why not? What about n houses and n utility centers?

Finding (Eulerian) Paths

Seven Bridges of Königsberg

1. Find the image showing the city of Königsberg and its bridges. Is it possible to walk a path through the city that crosses every bridge exactly once? Why or why not?

2. I want to walk a *loop* through the city that crosses every bridge exactly once and starts and ends in the same place. What is the minimum number of new bridges necessary to make this possible?

San Juan Islands Ferry Routes

3. Find the image showing the San Juan Islands ferry routes between Anacortes, Lopez Island, Shaw Island, Orcas Island, and San Juan Island (every red line between two of these locations is a ferry route). Is it possible to find a route in which you ride every ferry route exactly once? If not, explain.

4. Suppose the Washington State Department of Transportation miraculously has the funds to to add a ferry stop on Lummi island. Can you add three new routes to Lummi island so that it is possible to ride every ferry route exactly once? Why or why not?

5. Add three additional islands of your choosing to the ferry system and try to re-do the ferry routes so that you can make a loop that takes every route exactly once. What is the maximum number of routes you can have so that this is possible?

Graph Coloring

1. Is it possible to color each region of the Seattle city council district map with only three colors so that no two bordering regions share the same color? We call such a coloring a *proper coloring*.

2. What is the minimum number of different colors necessary to properly color all counties in the state of Washington? Why?

3. Create your own map that needs at least 4 colors for a proper coloring. What is the minimum number of necessary regions to accomplish this?

4. Suppose there were planet shaped like a torus (i.e. a donut) and had no oceans. Create a map on this planet that requires as many colors as possible to nicely color.