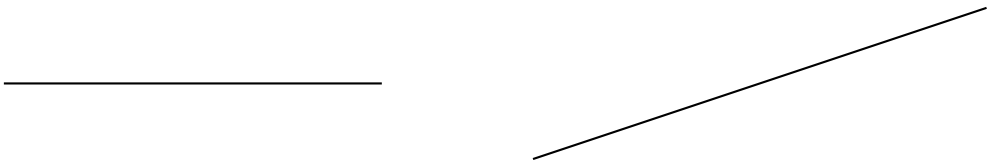
UW Math Circle

Week 23 – Geometric Constructions

Today we're learning about geometric constructions - your only tools are a pencil/pen, a compass for making perfect circles, and a straight edge to make perfectly straight lines. The compass is the only tool for measuring distances! To do so, set your compass to the distance between two points, then you can draw a new circle/arc with that radius located at a different point.

Let's practice with the first tools for geometric constructions:

- Copy the length of the following line segments.



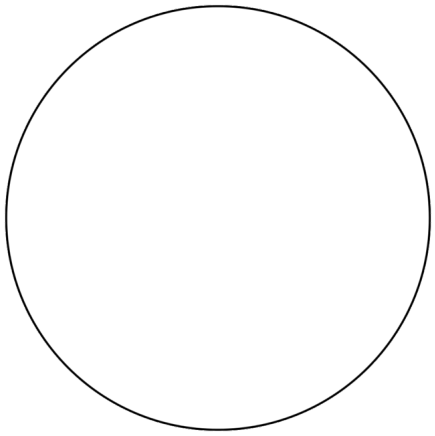
- Copy the following angles.

- Construct perpendicular bisectors (a line that splits the given segment into two equal parts and forms a right angle) for the given segments.

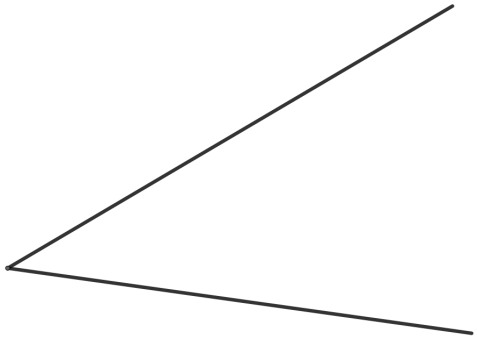
Now, try to figure out these next constructions on your own and justify why your construction works.

5. Construct a perfect equilateral triangle (a triangle where all the sides are the same length).

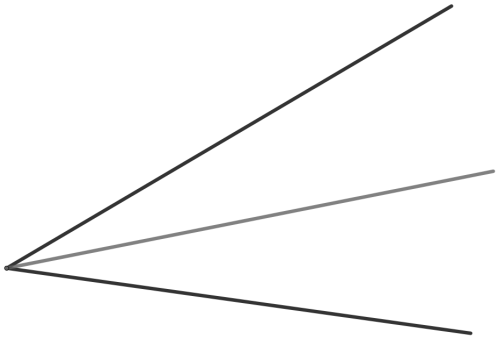
6. Find the exact center of the circle below.



7. Find the line that perfectly bisects the two lines below (a line that splits the angle below into two equal angles).



Perfectly bisect these two lines.



This is the goal.

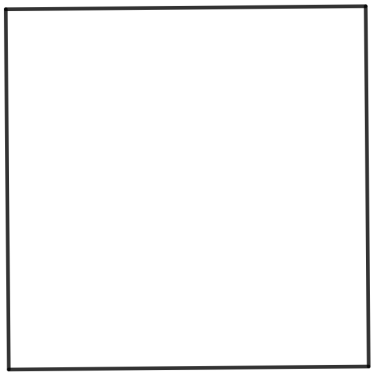
Stop here. Request the next page from your instructor when your group is done.

8. Construct a perfect square.

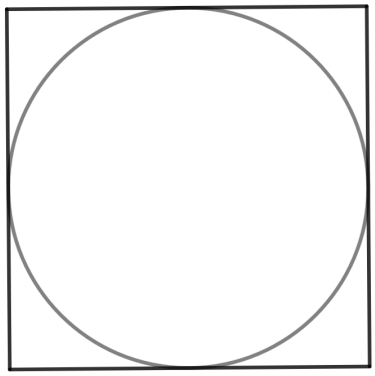
9. Construct a perfect hexagon.

10. Can you construct a 60° angle? What about 90° angle? Try to construct 30°, 45°, and 75° angles.

11. We say that a circle is inscribed in a square if it just barely touches each side, but doesn't cross any of the sides (see below). Perfectly inscribe a circle in the square below.



Inscribe a circle in this square.



This is the goal.

12. Can you inscribe a square in a circle? What about inscribing a circle in an equilateral triangle? What about an equilateral triangle in a circle?

13. Do the above constructions change if you start with the shape inside first? This process is called circumscribing, so a circle inscribed in a square looks the same thing as a square circumscribed about a circle; the only difference is that we start with the circle for the second!

Stop here. Request the next page from your instructor when your group is done.

Constructible Numbers

14. The line segment below is one inch long. Can you make a line segment of length 2? What about length 3? Length 1/2? What about length $\sqrt{2}$? What other lengths can you make?

15. Below are two segment, one with length a and the other with length b. Can you construct:

(a) A segment of length $a + b$?

(b) A segment of length $a - b$?

(c) A segment of length ab? (hint: similar triangles!)

(d) A segment of length $\frac{a}{b}$?

(e) A segment of length $\sqrt{a}$?

(f) A segment of length $\sqrt[3]{a}$?

16. We call a number d *constructible* if $|d|$ is the length of a line segment that you can construct (given the segment of length 1). For example, problem 15 showed $2, 3, \frac{1}{2}$ and $\sqrt{2}$ are all constructible! Of the following numbers, which do you think are constructible? Explain why!

    (a) -1

    (b) 15

    (c) $\frac{12}{13}$

    (d) $6\sqrt{3}$

    (e) π

    (f) $\frac{1+\sqrt{5}}{2}$

    (g) $\sqrt[3]{2}$

    (h) $\sqrt[4]{2}$

    (i) $\sqrt{\sqrt{10-\sqrt{7}}}$

17. Pick one of the numbers you decided was constructible and make a segment with that length! You first have to set what length 1 is.

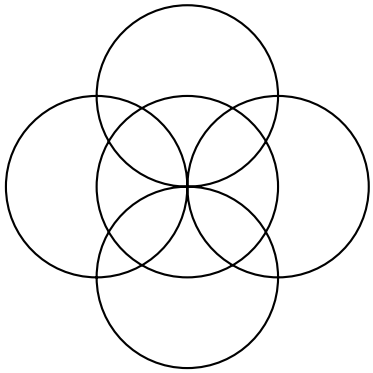
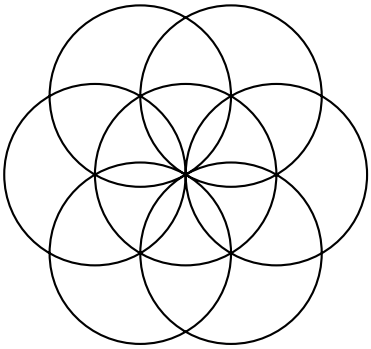
Stop here. Request the next page from your instructor when your group is done.

Constructing Tile Patterns

One use of constructions was making intricate tile patterns before modern technology existed. All of the tiles below were constructed using only a straightedge and a compass!



The two most common bases are perfect circles with either 4 or 6 secondary circles around them (we will call them 4-rosettes and 6-rosettes):

18. Can you construct both of these patterns?

The rosettes are part of the supporting guidelines for making tile patterns. In these next problems, draw the rosettes (and any other supporting guidelines) in pencil before inking over the final pattern in pen. Then you can erase your guidelines to reveal a clean final pattern!

19. Using the 4-rosette, can you construct a square circumscribing the center circle?

20. Divide the center circle into 8 equal parts. Can you find a way to divide it into 12 equal parts?

21. Using the 6-rosette, can you construct a hexagon inscribed in the center circle?

22. Divide the hexagon you constructed into 6 equal parts (through the center). Can you find a way to divide it into 12 equal parts?

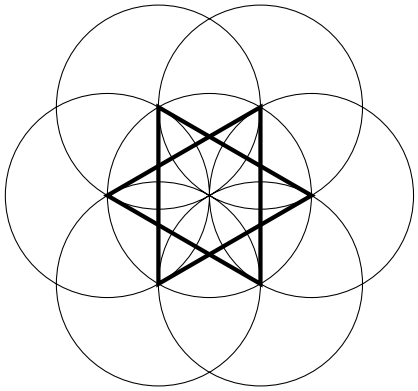
23. What other geometric shapes can you find using these two rosettes?

Stop here. Request the next page from your instructor when your group is done.

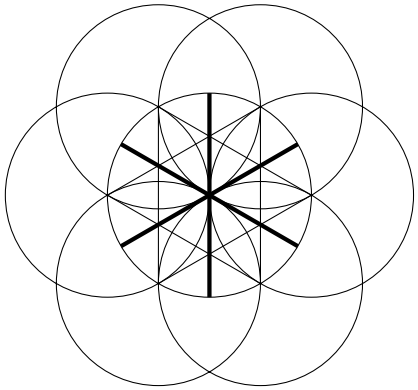
24. Here's a way to make a 12 pointed star:

    (a) Start with the 6 rosette.

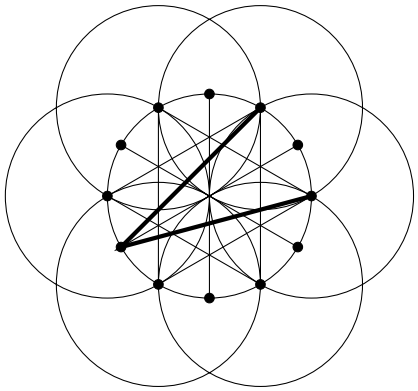
    (b) Connect every other point of the rosette to produce a six-pointed star. Notice there is a hexagon in the center of the star.



    (c) Draw a line through opposite corners of this hexagon, extend to the edge of the center circle



    (d) Mark the points on the circle, using your pen, connect every 5th point to make a star! The diagram just shows the first couple of points connected.

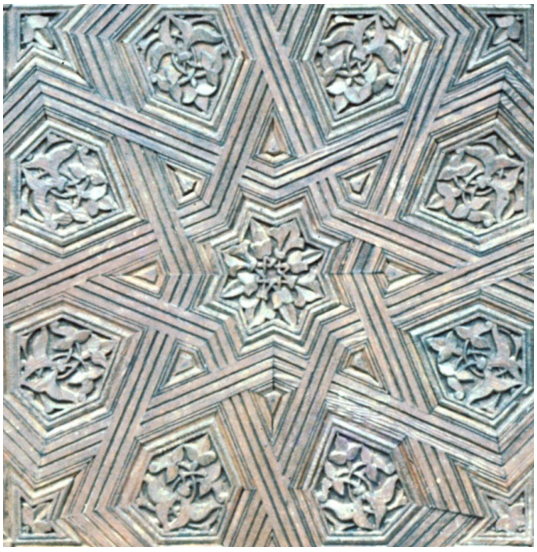


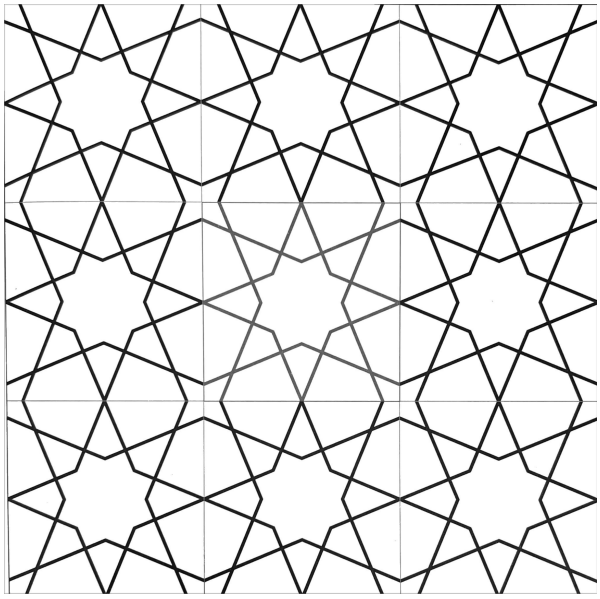
    Try to make one yourselves!

25. What other stars can you make? Try to find more than one for each rosette base.

Stop here. Request the next page from your instructor when your group is done.

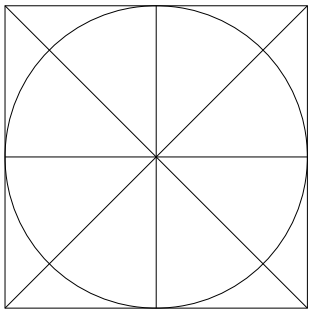
This tile pattern can be found in the Great Mosque of Cordoba, constructed in the year 784.



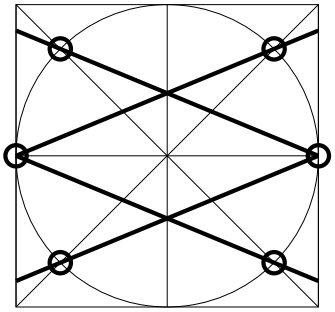


Let's make it!

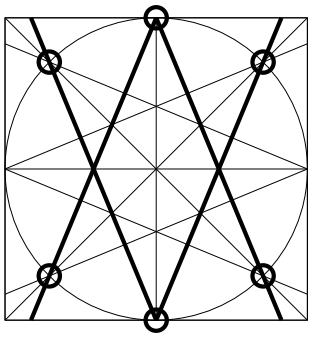
1. Using the 4 rosette, draw a circle in a square divided into 8 pieces



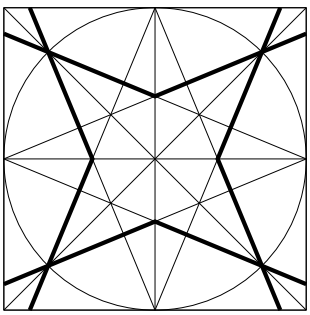
2. Connect the ringed intersections with four lines to create two interlocking 'V' shapes.

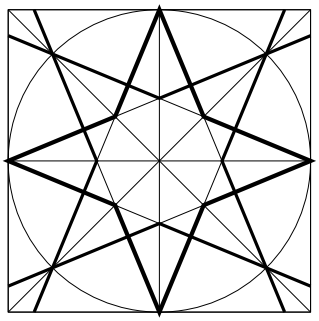


3. Create another set of interlocking ‘V’s as shown.

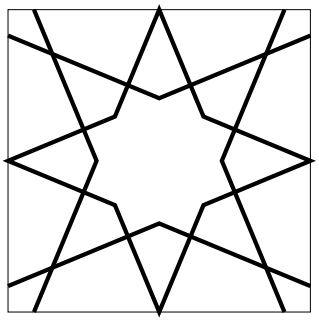


4. Using your pen, ink over the indicated lines.

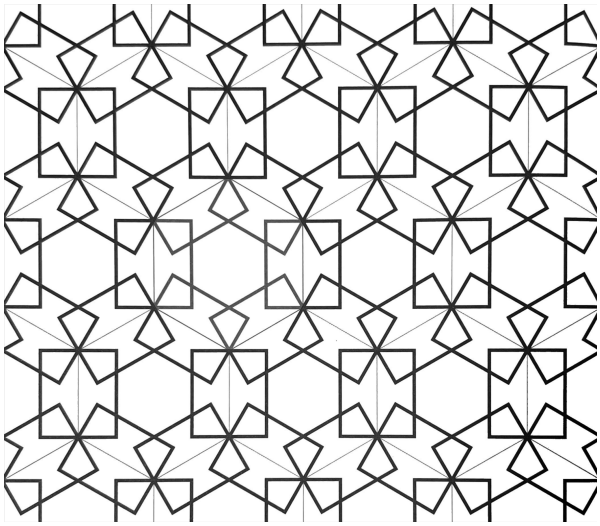




5. We’re done! Erasing the construction lines will reveal the final pattern.

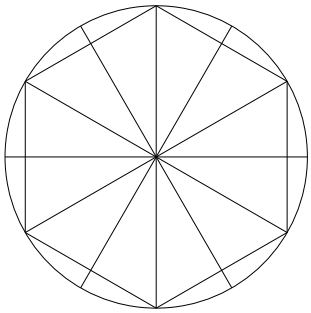


This tile pattern can be found in the 'Abd al-Samad Complex, built in the year 1304.

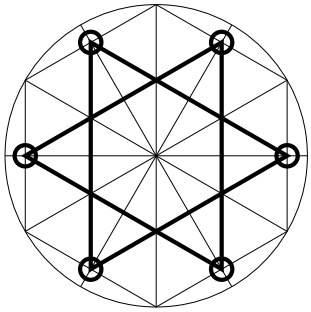


Let's make it!

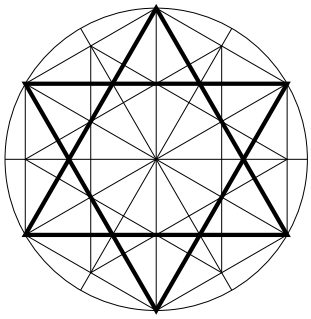
1. Using the 6 rosette, draw a hexagon in a circle, divided into 12.



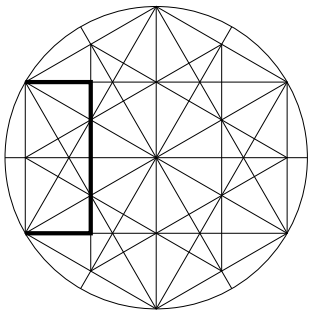
2. Draw two triangles within the hexagon using the marked intersections as guides.



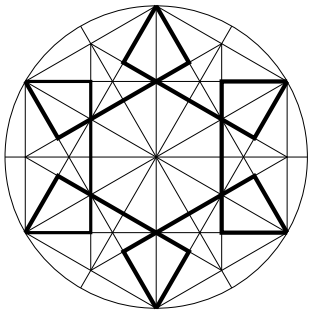
3. Draw another 2 triangles as shown.



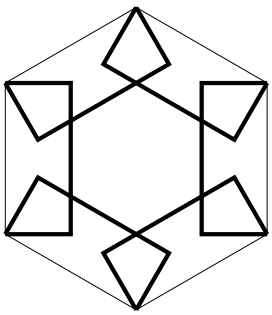
4. Ink over the bold lines in pen.



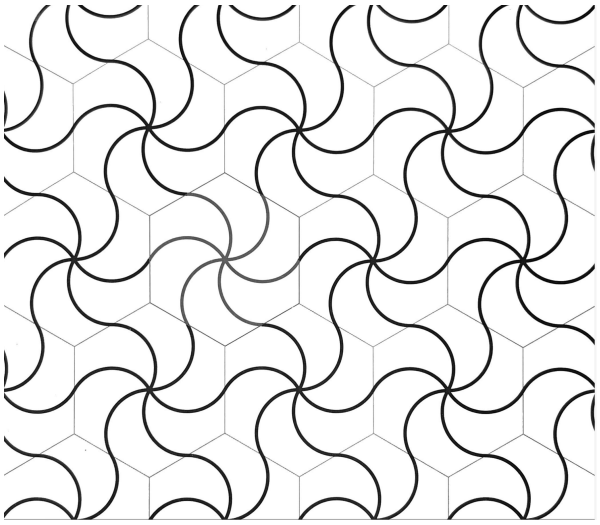
5. Repeat the shape from step 5 a further 5 times.



6. We're done! Erasing the construction lines will reveal the final pattern.

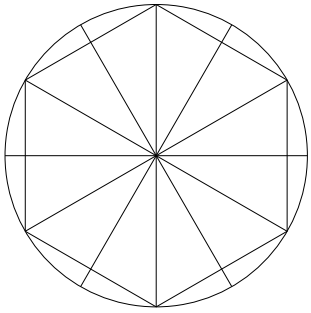


This tile pattern can be found in the Alhambra, built sometime between 1302 & 1391.

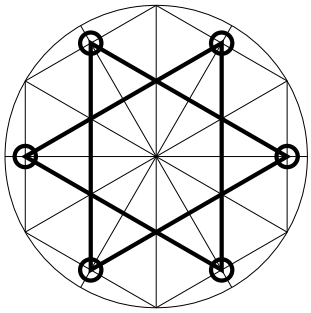


Here's how to construct it!

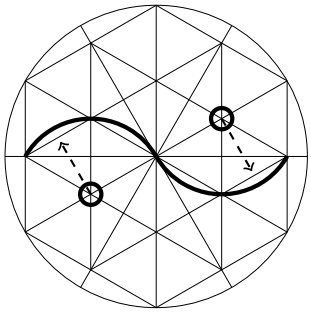
1. Using the 6 rosette, draw a hexagon in a circle, divided into 12.



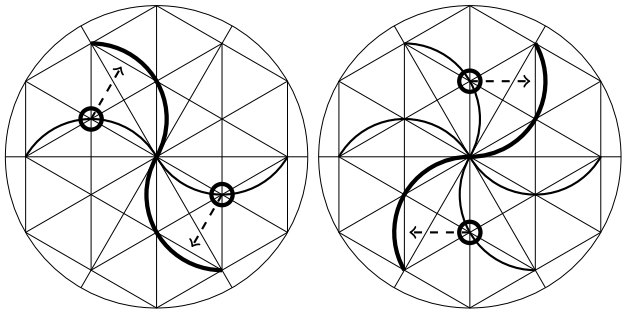
2. Used the ringed intersections as markers to draw two triangles as shown.



3. Using a pen, put your compass on the ringed intersections and make the curves shown.



4. Place the compass point on the ringed intersections at the top of the first arc and the bottom of the second arc to make a second pair of arcs. Draw the third pair of arcs in the same way.



5. You're done! Erasing the constructions will reveal the pattern.