Name: _____

UW Math Circle

Week 16 – Moving Furniture

1 The Moving Sofa Problem

What's the largest sofa you can fit through a one-meter-wide hallway with a 90° turn?

You could have a 1m × 1m square sofa, for an area of 1m^2. Or better yet, you could have a semicircular sofa with radius 1m, for an area of $\frac{\pi}{2} \approx 1.57\text{m}^2$.

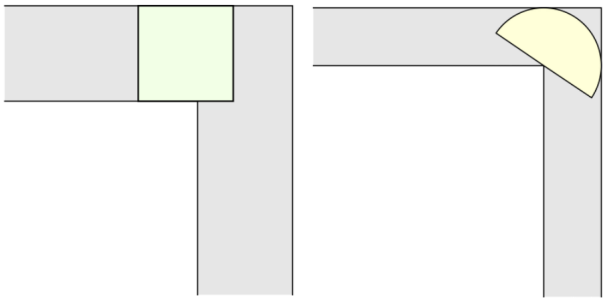
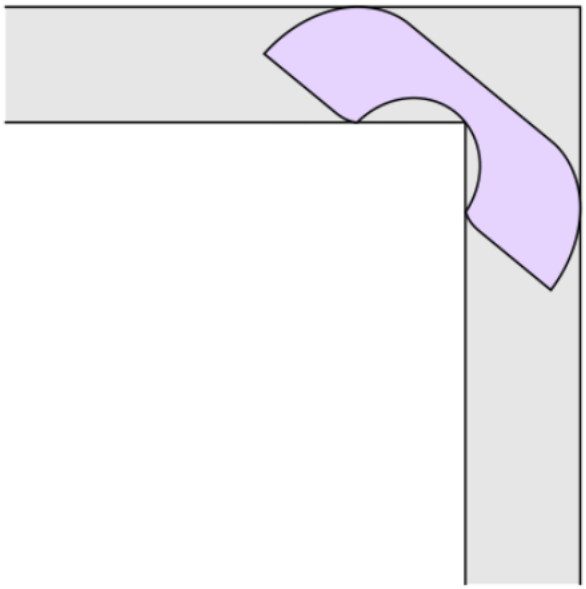


Image source: https://www.math.ucdavis.edu/ romik/movingsofa/

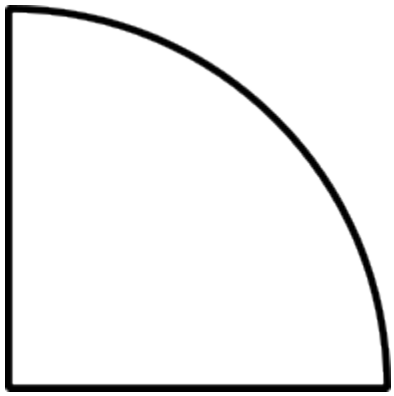
The largest possible sofa is this one, with area $\sim 2.22\text{m}^2$. Joseph Gerver discovered this sofa shape in 1992, and Jineon Baek proved that it's the largest in 2024! ← Exclamation point, not factorial.



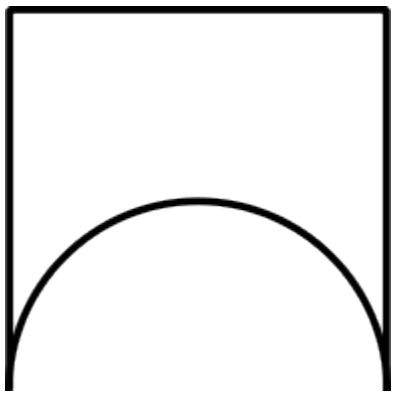
Let's practice finding some areas. Remember that the **area of a circle with radius** r is

$$\text{area} = \pi \cdot r^2.$$

1. This sofa is a quarter circle with radius one meter. What's its area?

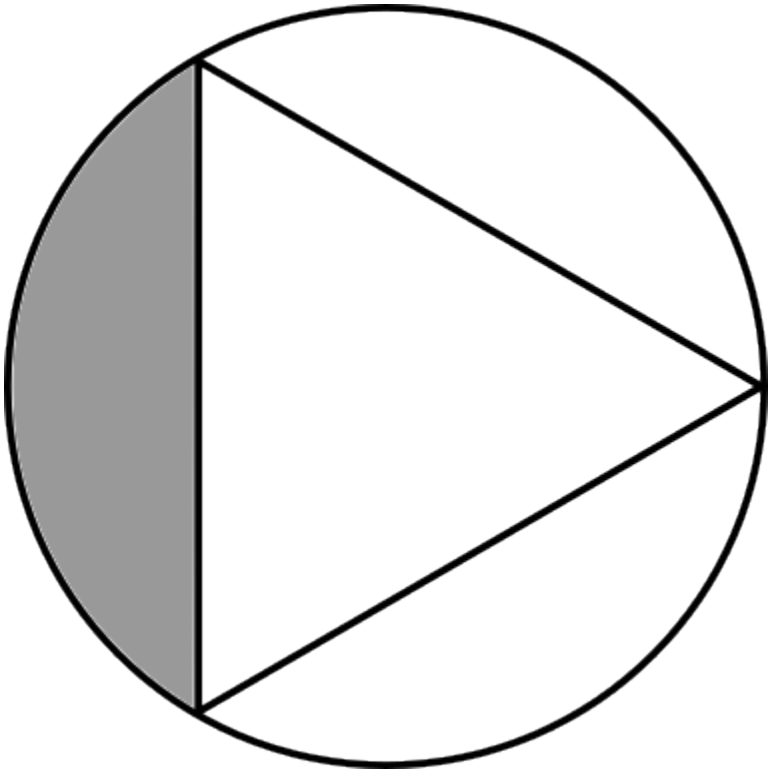


2. How about this sofa, a one-meter-by-one-meter square with a semicircle cut out?

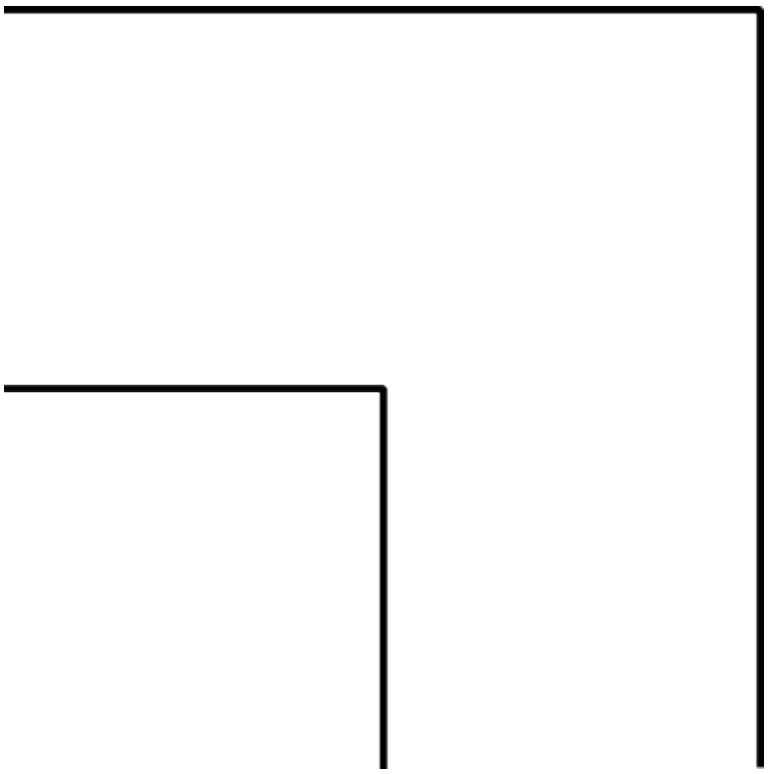


3. An equilateral triangle is inscribed in a circle of radius one meter. The sofa is just the shaded part. What's the area of the sofa?

   Hint: Find the area of the circle and triangle first! The area of a triangle is $\frac{1}{2} \cdot \text{base} \cdot \text{height}$.



4. Here's the hallway from before, one meter wide with a 90° turn. What's the largest sofa you can fit through the hallway *without rotating it*? That is, you can move the sofa horizontally or vertically, but you can't turn it at an angle. How do you know your sofa is the largest possible?

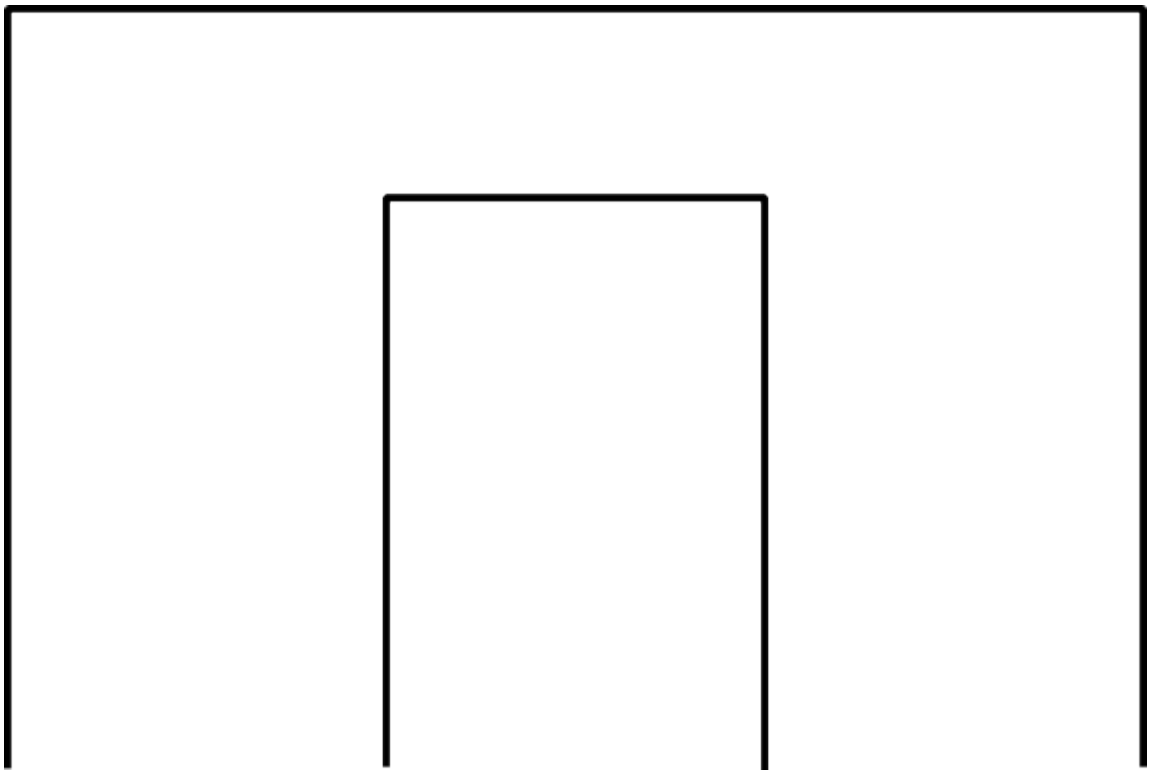


Here are some differently shaped hallways. Same question: **what's the largest sofa you can fit through the hallway?** This is what you'll have to do:

i. Cut out your "sofa", and make sure it fits through the hallway without going over the sides. Remember, your goal is to make the largest sofa possible!

ii. Find the area of your sofa. Assume the hallway is one meter wide at the beginning and end. If you don't know how to calculate the exact area, you can trace the sofa on your graph paper, and estimate its area by counting the squares inside. Each square represents $\frac{1}{64}\text{m}^2$.

5. What's the largest sofa you can fit through this hallway? The horizontal part is 0.5 meters wide.



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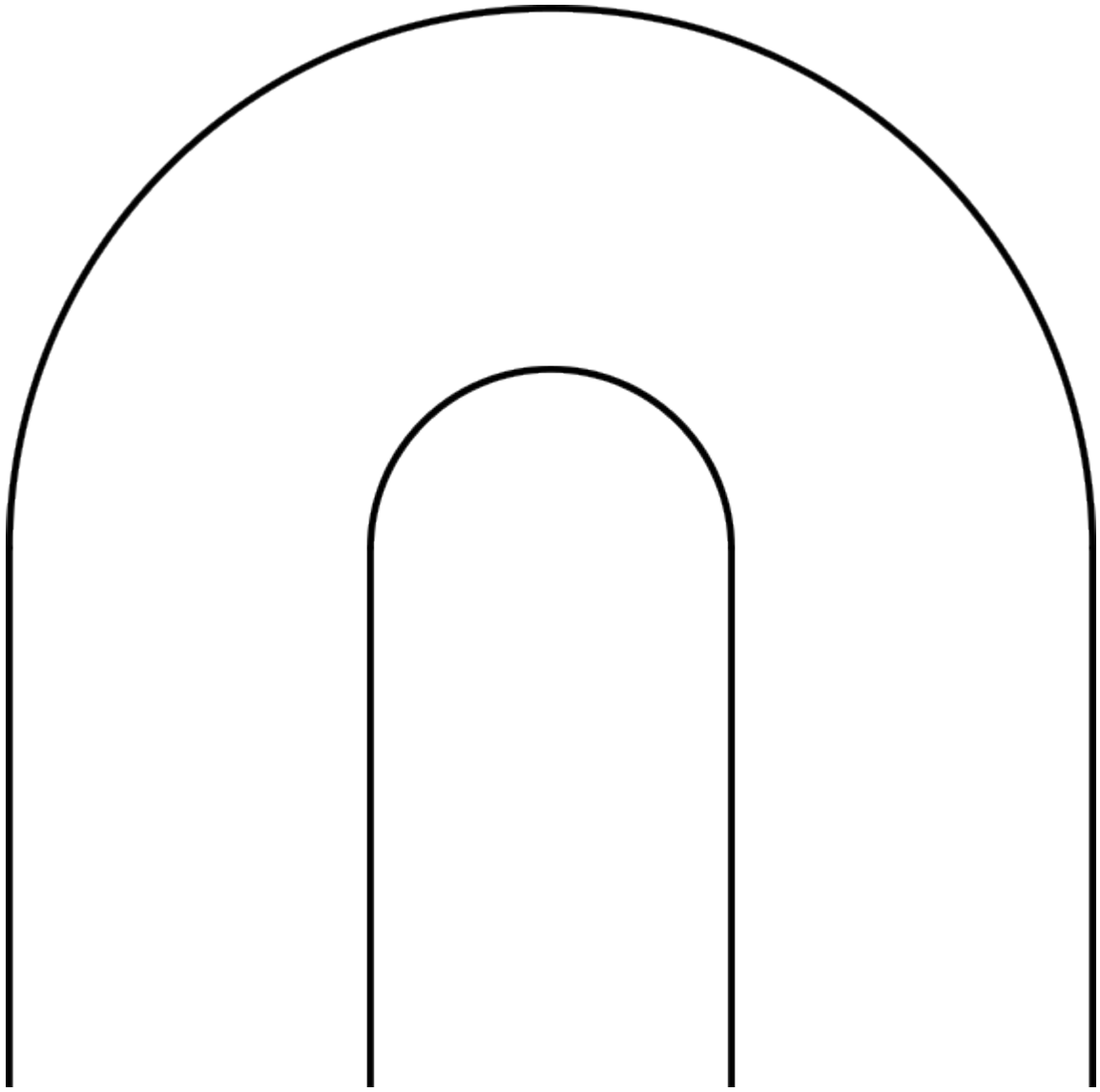

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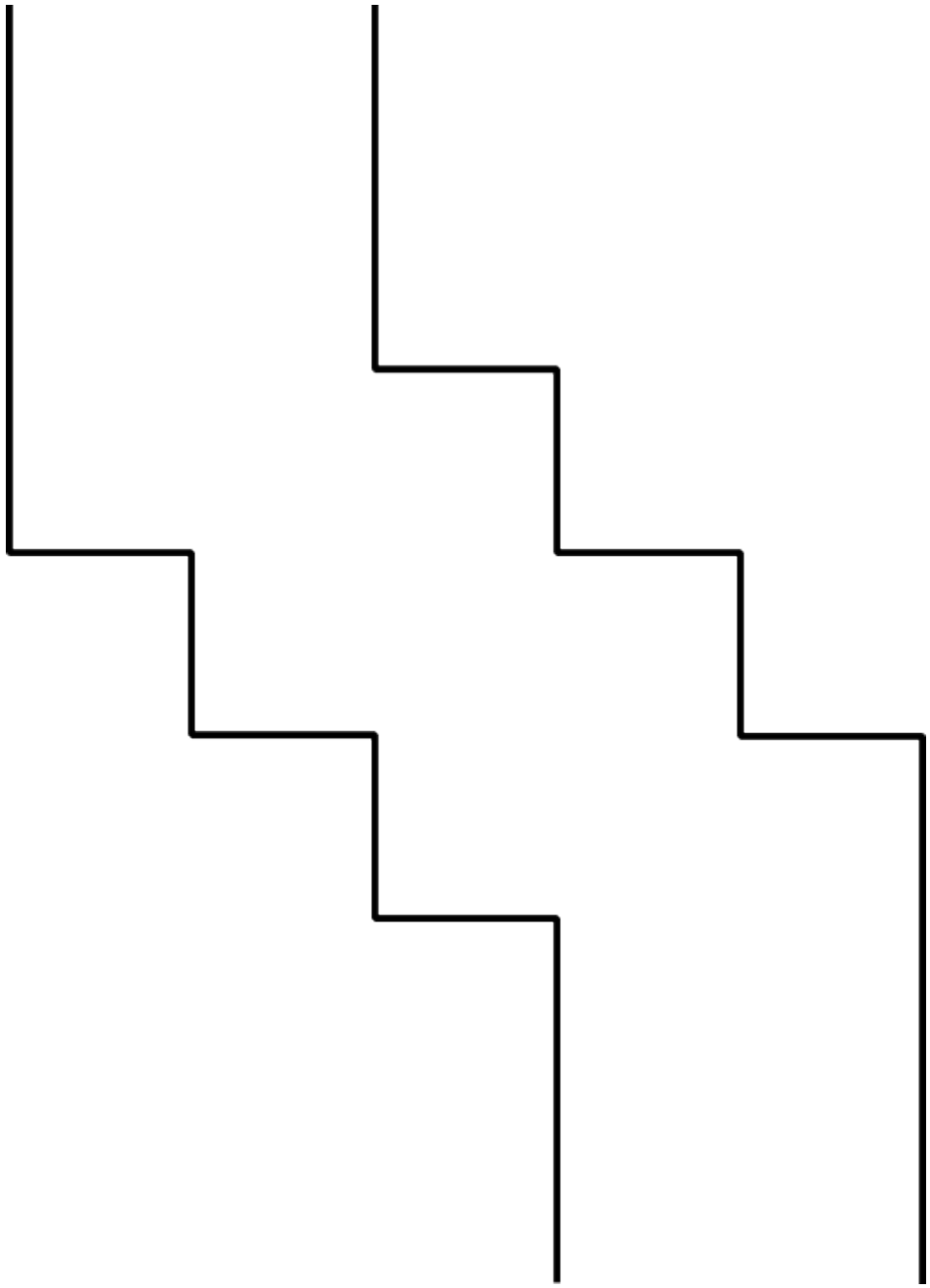

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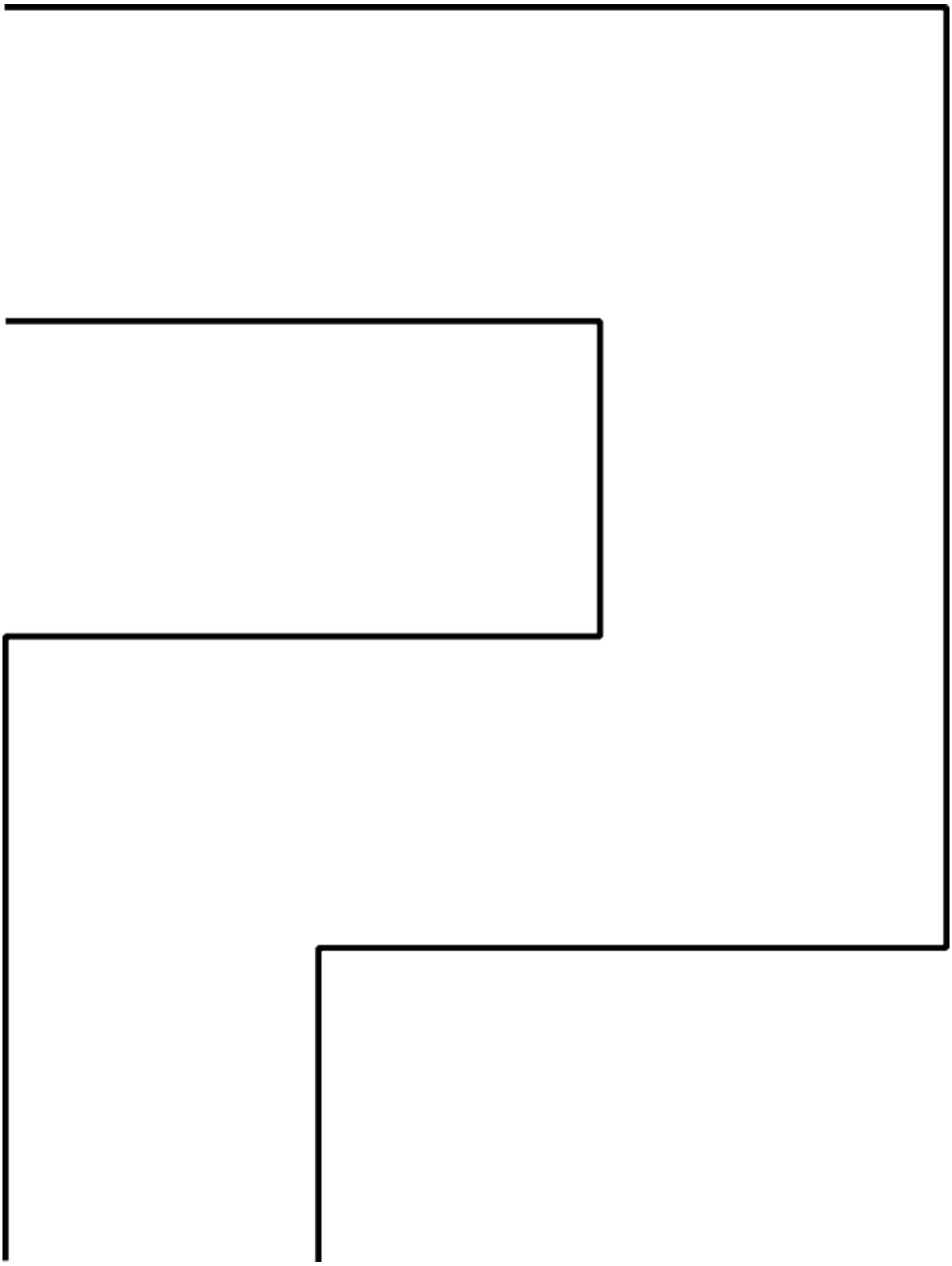
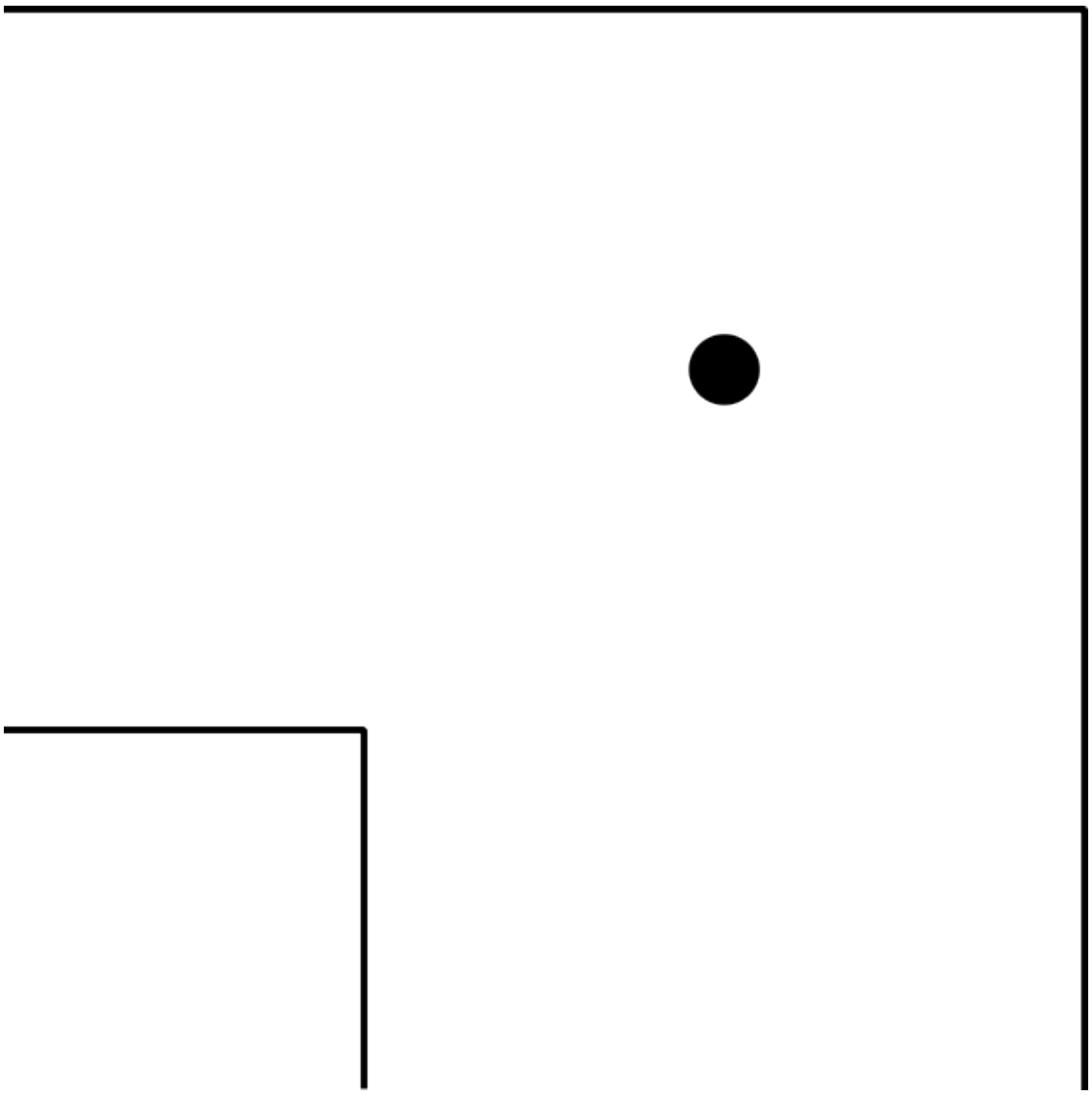


9. This hallway is two meters wide, but there's an inconvenient pole blocking your way.



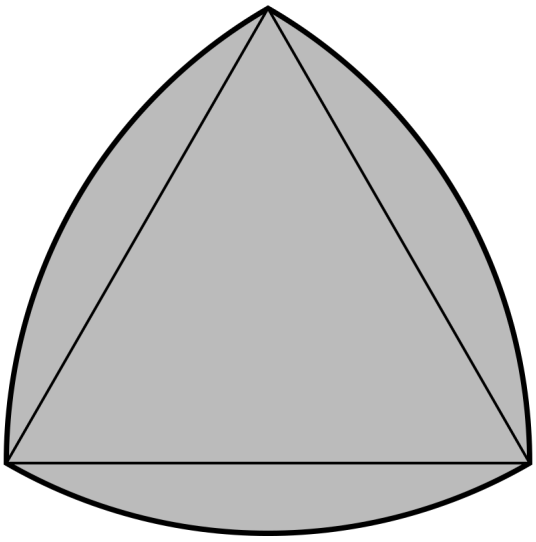
2 More Navigation Problems

10. Now that you've moved your sofa, you need to carry a *lamp* through the original, one-meter-wide hallway with the 90° turn. The hallway ceiling is very low, so you have to carry the lamp horizontally (think of it as a line segment). What's the longest lamp you can fit through the hallway? How do you know?

11. What's the longest lamp you can fit through each hallway from problems 5-9?

12. You're looking for a room to practice morris dancing. Your dance routine involves holding a one-meter-long stick horizontally and rotating it 360°. It's okay if the center of the stick moves, so long as the stick makes a full 360° turn.

    What's the smallest room you could practice in? One option is a circular room with diameter one meter (area $\frac{\pi}{4} \approx 0.785\text{m}^2$). But the room below, with corners one meter apart, also works and is actually smaller (area $\sim 0.705\text{m}^2$). Is it possible to do even better?



    Hint: Try drawing the room and using your pencil as the stick! You can estimate the area with your graph paper if needed.