UW Math Circle

Week 4 – Polyhedra!

How many **vertices** and **faces** can a polyhedron have?

Build some polyhedra! When you build a new polyhedron, sketch it where it belongs in the table. For example, if your polyhedron has 8 vertices and 6 faces, sketch it in row 8 and column 6. **Fill out as much of the table as you can.** Be warned—some numbers might be impossible!

Vertices \ **Faces**	4	5	6	7	8	9	10
4							
5							
6							
7							
8							
9							
10							

Presenting Tips:

1. Start by stating the problem in your own words, and summarize the solution you're about to present.

2. Explain each step clearly and justify everything you say. We should be able to follow your solution even if we haven't worked on the problem ourselves.

3. If something's important, write it down! Write full equations (eg. "$x = 10$", not just "10"). Label your diagrams clearly.

4. Talk to your audience! Face the room, and speak up so everyone can hear you.

Tell an instructor when you're ready to present a solution! Once your group has presented Problems 1 and 2, you'll receive a new worksheet.

Euler's Formula

If a polyhedron has V vertices, E edges, and F faces, **Euler's formula** says that

$$V - E + F = 2.$$

1. Günter builds a polyhedron out of 20 triangles, 30 squares, and 12 pentagons. **How many edges and vertices does the polyhedron have?**

   Hint: Count the edges first! If you add up the sides of every polygon, how many times have you counted each edge?

2. Prove that it's impossible to build a polyhedron out of 20 squares and 25 triangles.

Comparing Faces

3. A polyhedron has V vertices, E edges, and F faces. **Prove that E is at least $\frac{3F}{2}$.**

   Hint: What's the least number of sides a single face can have?

4. **Prove that E is at least $\frac{3V}{2}$.**

   Hint: What's the least number of edges that can meet at a single vertex?

5. **Prove that F is at most $2V - 4$.**

   Hint: Rewrite Euler's formula as $E - F = V - 2$, then use Problem 3.

6. **Prove that V is at most $2F - 4$.**

   Hint: Rewrite Euler's formula as $E - V = F - 2$, then use Problem 4.

Use Problems 5 and 6 to explain what we saw in our table!