

Roles:

1. Marker Wielder - Responsible for writing work on the white board or on the page.
2. Presenter - Responsible for presenting work to an instructor.
3. Leader - Responsible for helping the Presenter practice and calling over an instructor when ready to present.

After finding and presenting a solution to an instructor, you will be given a new problem! Switch roles when a new problem is received.

Presenting Tips:

1. Start by stating the problem in your own words, and summarize the solution you're about to present.
2. Explain each step clearly and justify everything you say. We should be able to follow your solution even if we haven't worked on the problem ourselves.
3. If something's important, write it down! Write full equations (eg. " $x = 10$ ", not just "10"). Label your diagrams clearly.
4. Talk to your audience! Face the room, and speak up so everyone can hear you.

(Flip this page over!)

As a group, fill in the missing pieces of the following proof!

Cards

A deck contains 15 cards: 5 hearts, 5 spades, and 5 clubs. You draw cards from the deck one at a time without looking. How many cards must you draw to guarantee that you have at least one card of each suit?

Proof. If you draw at least _____ cards, then you are guaranteed to have at least _____ card in each suit because the first _____ suits only have _____ cards total.

If you draw at most _____ cards, then you are **not** guaranteed to have at least _____ card in each suit because it is possible to have _____ cards from the first _____ suits and _____ cards from the last suit.

Therefore, you must draw _____ cards to guarantee having at least _____ card in each suit. □

Soccer Sportsmanship

A soccer team of 11 players just won a big game. After the game, some (but not all) pairs of players give each other high-fives to celebrate. Show that there are two players on the team that gave the same amount of high-fives.

Proof. Suppose for the sake of contradiction that _____

(Put the opposite of what you're trying to prove here!)

Then, the numbers of people each person high-fived must be

_____, _____, _____, _____, _____, _____, _____, _____, _____, _____, and _____.

But then one of the players on the team must have high-fived

_____, while another one of the players on the team

must have high-fived _____. This is impossible,

because _____

_____!

Therefore, there must have been two players on the team that gave the same number of high-fives.



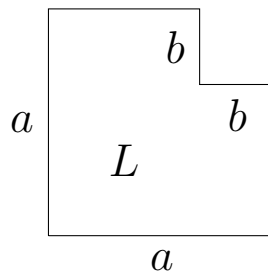
Formula: The area of an $a \times b$ rectangle is $a \cdot b$.

Difference of Squares

Prove

$$a^2 - b^2 = (a + b)(a - b)$$

visually when $a > b > 0$.

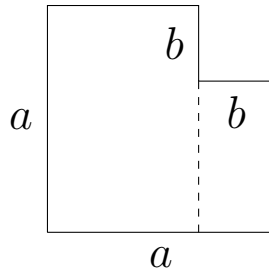


Proof. Cut out a large square and label its sides with a .

Then, cut out a smaller square from the top right corner of the $a \times a$ square and label the cut sides by b .

It is possible to cut a $b \times b$ square from the $a \times a$ square because $a \underline{\hspace{1cm}} b$.

The $a \times a$ square has area _____, and the $b \times b$ square has area _____. So, the area of L is _____.



Now, cut the shape vertically into two rectangles.

Take the remaining rectangles and line them up to make one big rectangle.

The width of the big rectangle is _____ and the height is _____,
so the area is _____.

Since the area of L is _____ to the area of the big rectangle, we
can conclude that

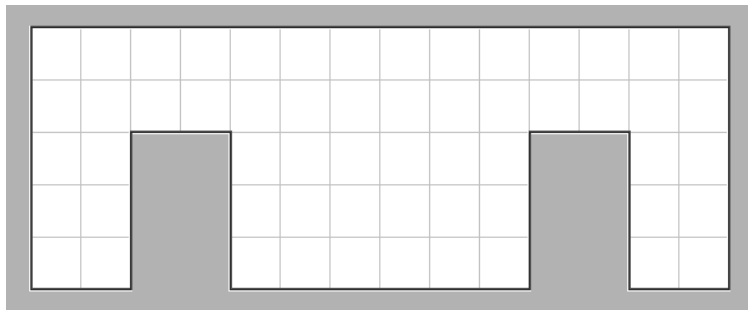
$$\text{_____} = \text{_____}.$$



In the Hall of the Mountain King

The King of the Mountain Dwarves wants to light his underground throne room (shown in white in the image below) by placing several torches so that the whole room is lit. The king, being very miserly, wants to use as few torches as possible. What is the least number of torches he could use? The walls in the room are lined with velvet and do not reflect light.

(You also should show why he can't do it with a smaller number of torches!)



99-gon

Each vertex of a 99-sided polygon is colored red or blue. Prove that there must be two adjacent vertices with the same color.

Battleship

Sherlock and Mycroft are playing Battleship on a four-by-four grid. Mycroft hides a single three-by-one cruiser somewhere on the board. Sherlock can pick squares on the grid and fire upon them. What is the smallest number of shots Sherlock has to fire to guarantee at least one hit on the cruiser?

(Show also that Sherlock can't do it with a smaller number of shots!)

Talent Show

The University of Washington is holding a talent competition. The competition has five contests: acrobatics, basketball, chemistry, dancing, and exercising. Any student can enter into any number of the contests but only once for each one. It turned out that each student participated in an odd number of contests and each contest had an odd number of participants. Was the total number of contestants odd or even?

