

UW Math Circle

Week 10 – Puzzles

1. You are in a 100 story building and you have two identical marbles. You want to find out the highest floor in the building you can drop marbles off without them breaking. After you drop a marble, if it breaks it is unusable. If it survives it is not weakened at all.

Find the minimum number of drops you need to guarantee finding the highest floor you can safely drop marbles from.

For example, if you only had one marble: The only way to find the highest safe floor without fail is to start at floor 1 and go up one floor at a time, dropping it at every floor. (Bonus part: why?) In the worst case when the highest safe floor happens to be floor 100, you will use 100 drops. When you have two marbles, you can do much better!

2. You have 5 bags of 5 marbles each. All of the marbles in all of the bags are identical 1 gram marbles, except for one of the bags, which contains marbles that all weigh 1.001 grams. You have a digital scale that displays the numeric weight to an accuracy of 0.001 grams. What is the minimum number of weighings required to identify the bag with the slightly heavier marbles?

3. You want to stay at an inn for seven nights. You only have a straight chain (not circular) with seven gold rings linked together. The innkeeper agrees to accept this in payment for the stay, but you only want to pay for one night at a time.

By cutting a slot in a ring, you can remove it from the chain to pay the innkeeper. You start to cut a ring, but the innkeeper wants you to minimize the damage to the chain. What is the minimum number of cuts you must make, provided the provided the innkeeper is happy to return previously given links as change?

For instance, let's say you gave the innkeeper one ring on the first night and one on the second night. On the third night, you could pay with a chain of three rings, and he'd give you back the first two rings as change.

4. You have 8 pancakes in a stack, one with one blueberry, one with two blueberries, one with three blueberries, and so on. The pancakes are thin and you can always see all of the blueberries. Count the number of blueberries in the top pancake and call that number N . Pick up the stack of the top N pancakes and flip it upside down. Repeat this procedure until the pancake with one blueberry ends up on the top. Find an initial arrangement of pancakes that takes the largest number of flips for the pancake with one blueberry to end up on the top.

5. You need to transport 3000 apples from Wenatchee, WA (the “apple capital of the world”) to Las Vegas, 1000 miles away. Your truck can carry 1000 apples at a time, but you need to consume 1 apple for every mile you travel away from Wenatchee (you don’t consume anything traveling back towards Wenatchee). How many apples could you bring to Las Vegas? (You may drop some apples off at the side of the road and pick them up later.)

6. You’re exploring an infinite, flat desert in your dune buggy. You’ve heard of a river that runs through the desert in an infinite, straight line, but you don’t know where the river is, or which direction it runs. Also, the river is in a canyon, so you can’t see the river until you reach it.

You encounter a sign in the sand. It reads, “River: 10 km. →”. The sign has fallen down, so you can’t tell which direction it was pointing. Starting at the sign, what’s the shortest path you can drive to guarantee you’ll find the river?

7. There is a square plot of land with 1 km long sides. A perfectly straight powerline runs underneath through the plot, but we don't know where it enters or exits. You want to find any part of this powerline by digging as little distance as possible.

For example, you could dig around the whole perimeter (4 km) to guarantee finding the powerline. Can you do better? What is the best strategy you can find?