UW Math Circle

Week 8 – Cellular Automata

A *cellular automaton* is a grid of colorful squares that changes over time based on certain rules. To help us study cellular automata, we'll use a program called Golly (`golly.sourceforge.io`).

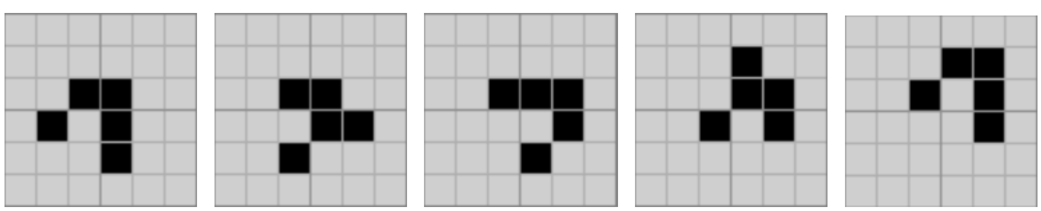
Golly tips:

- to advance the simulation one step
- Enter to start/stop the simulation
- + − to speed up/slow down the simulation
- [ ] to zoom
- Ctrl + R to reset
- Ctrl + I to read information

1 Conway's Game of Life

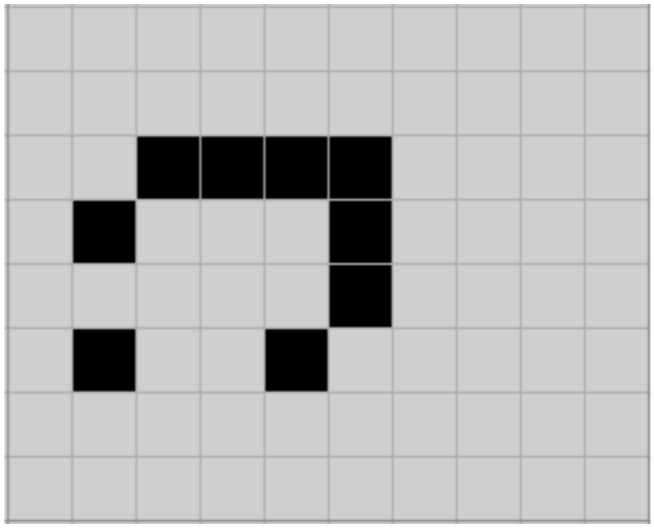
Conway's Game of Life is the most famous cellular automaton. Each cell in the grid is either alive (black) or dead (white). Here are the rules:

- If a dead cell has *exactly three* live neighbors (including diagonally!), it will come alive on the next step. Otherwise, it will stay dead.
- If a live cell has *two or three* live neighbors, it will remain alive on the next step. Otherwise, it dies.

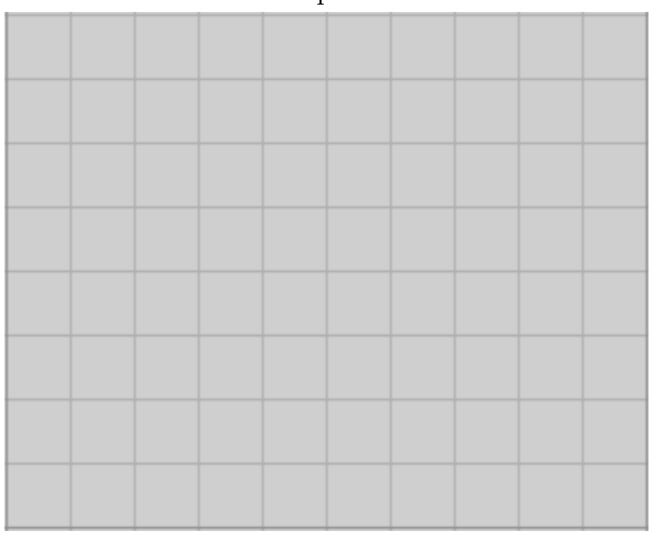


Step 0 Step 1 Step 2 Step 3 Step 4

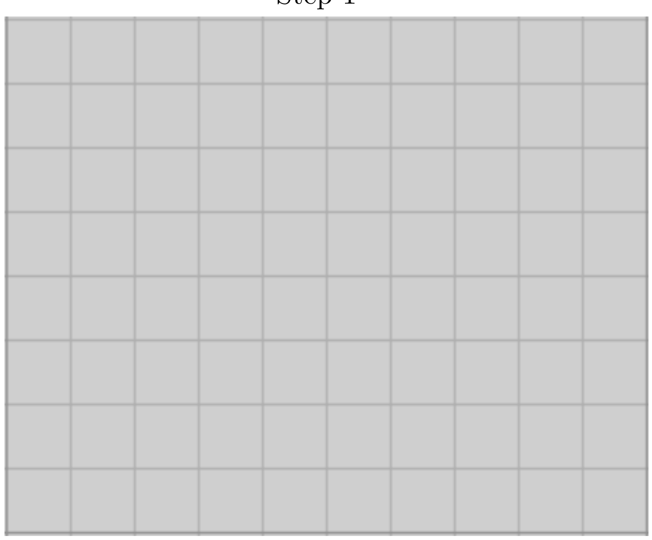
1. Try drawing the next two steps of the simulation on page 2. Then, use Golly to check your work!



Step 0



Step 1



Step 2

2. A *spaceship* is any pattern that can move through empty space on its own. See `Patterns\Life\Spaceships\spaceship-types.rle` in Golly for the many different spaceships in the Game of Life!

   You can calculate a spaceship's horizontal or vertical *speed* by waiting until it looks like it did initially, then dividing **(distance moved)/(number of steps)**. For example, the spaceship on page 1 takes four steps to look like it did initially, and in that time it moves on cell up and one cell to the right. So its horizontal and vertical speed are each 0.25 cells per step.

   (a) Explain why the pattern in Problem 1 is a spaceship. How many steps does it take to look like it did initially (i.e. an exact copy of the starting pattern, shifted to a different position)?

   (b) What's the horizontal speed of the spaceship from Problem 1, in cells per step?

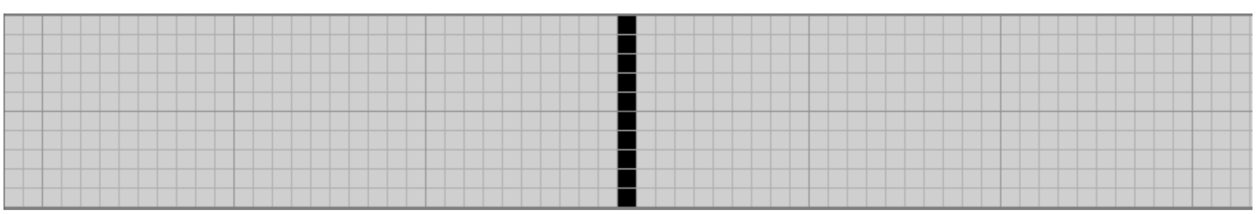
   (c) What's the maximum possible speed for a spaceship in the Game of Life (on an infinite, flat grid)?

3. We don't always have to play the Game of Life on a flat grid! Imagine the grid below rolled up like a tube, so the top row is adjacent to the bottom row. To create this in Golly:

   i. Open the File menu and select "New Pattern".

   ii. Open the Control menu and press "Set Rule".

   iii. Type "B3/S23:T0,10" and hit "OK".



   (a) What happens if you start the simulation with a vertical line? *****SEIZURE WARNING: this simulation can create a "flashing light" effect. Don't view this simulation in Golly if you're sensitive to flashing lights!**

   (b) Can you create a spaceship with a horizontal speed of one cell per step? *Hint: start with a vertical line, and try modifying the left side so it only moves to the right.*

   (c) Is it ever possible for a spaceship to move faster than one cell per step? Why or why not?

2 "Life-Like" Cellular Automata

Let's change the rules! "Life-like" cellular automata are rules similar to the Game of Life, where a cell's state (alive or dead) changes based on how many live neighbors it has.

The rules for the Game of Life are denoted **B3/S23**. This stands for "Born 3" (a dead cell comes alive if it has three live neighbors), "Survive 2, 3" (a live cell survives if it has two or three live neighbors). Another Life-like cellular automaton might use the rule **B234/S**: a dead cell becomes alive if it has two, three, or four live neighbors, while a live cell always dies.

You can experiment with different rules in Golly using "Set Rule" in the Control menu.

4. How would you write the rule for a Life-like cellular automaton that never changes from the starting pattern?

5. Open `Patterns\Life-Like\Day-and-Night-gun-and-antigun.rle` and run the simulation. This is a Life-like cellular automaton with the rule **B3678/S34678**.

   (a) Notice that the "spaceship launcher" patterns on the left and right are identical, with colors inverted, and they behave exactly the same way.

   In general, imagine taking any starting pattern and creating a new pattern by inverting all the colors of the original. When you run both the original and inverted versions, will the two simulations always remain color-inverted copies of each other? Why or why not?

   (b) A Life-like cellular automaton has *C-symmetry* if the answer to part a is "yes". Can you write the rule for another cellular automaton with C-symmetry? Does the Game of Life have C-symmetry?

   (c) How would you write the rule for a color-inverted version of the Game of Life?

6. Open `Patterns\Life-Like\replicator.rle` and advance the simulation step by step. OK! OK! OK! OK! OK! This is a Life-like cellular automaton with the rule **B1357/S1357**.

   Will this cellular automaton always replicate its starting pattern? Why or why not?

7. Can you find a Life-like cellular automaton with a spaceship that moves one cell per step, without needing a modified grid as in Problem 3?

3 Loops

Now, let's look at another type of cellular automata! These ones have more complex rules, and the cells have more than two possible states.

8. Open `Patterns\Loops\SDSR-Loop.rle` and run the simulation. Pause and advance it step-by-step so you can see what's going on.

   Each loop has a series of light-blue and yellow cells circulating around it. How do the yellow cells behave differently from the light-blue ones? What role do the light-blue and yellow cells play in the loop's reproduction?

9. Open `Patterns\Loops\Evoloop-finite.rle` and run the simulation. Slow it down at the times specified and answer the questions below.

   - **Step 8000-10000:** Can you spot a new type of loop? What causes these loops to replicate differently from the others?

     Is the new type of loop surviving better than the original, or not as well? Why?

   - **Step 18000-20000:** Can you find yet another type of loop? How is this type different from the first two?

     How is it possible for new types of loop to appear, when the original loops were "programmed" to copy themselves exactly?

   - **Step 30000-32000:** How many different types of loop can you see? How many loops of the original type are left?

     What do you predict the simulation will look like at step 100000, and why?

   - **Step 100000:** Was your prediction correct? Why or why not?

     How is this cellular automaton related to the theory of evolution and natural selection in biology?

10. (Open-ended questions.) You've now seen patterns in cellular automata that move, grow, reproduce, adapt, and evolve over time.

    - In a far more complex cellular automaton, do you believe there could be more advanced "life forms" that think or make decisions? Why or why not?
    - **If you said no:** our universe is like a cellular automaton in some ways—it's made of tiny "cells", or particles, and it follows mathematical rules. In your opinion, what are the main advantages of our universe for supporting advanced life, as opposed to a cellular automaton?
    - **If you said yes:** suppose you ran this cellular automaton on your computer, and advanced life forms developed inside. If you stopped the simulation, would you be killing them? Would you be morally obligated to keep the simulation running indefinitely?